TRANSITION IN BOUNDARY LAYER FLOWS

by

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Boundary Layer Wind Tunnel Facility
DECLARATION

I hereby declare that the following work has been composed by myself and that this dissertation has not been presented for any previous award of the C.N.A.A. or any other University.

Iain D Gardiner
Transition in Boundary Layer Flows

by

Iain D Gardiner

Abstract

An experimental investigation of transition in boundary layer flows under the influence of various freestream conditions is described.

Velocity profiles are obtained automatically by means of a stepper-motor driven traverse mechanism which carries a hot wire probe connected to a constant temperature anemometer and associated instrumentation. This was achieved by use of a data acquisition and control facility centred around a microcomputer with a Eurocard rack mounted extension. The automatic boundary layer traverse is software controlled and the data obtained is stored in a disc file for subsequent analysis and graphical display. As an integral part of this facility a successful method of obtaining reliable intermittency values from a hot wire signal was developed.

The influence of freestream turbulence and pressure gradient upon transition within a boundary layer developing on a flat plate is elucidated by a series of controlled experiments.

From the data accumulated, the concept of statistical similarity in transition regions is extended to include moderate non-zero pressure gradients, with the streamwise mean intermittency distribution described by the normal distribution function.

An original correlation which accounts for the influence of freestream turbulence in zero pressure gradient flows, and the combined influence of freestream turbulence and pressure gradient in adverse pressure gradient flows, on the transition length Reynolds number $R_\theta$, is presented. (The limited amount of favourable pressure gradient data precluded the extension of the correlation to include favourable pressure gradient flows).

A further original contribution was the derivation of an intermittency weighted function which describes the development of the boundary layer energy thickness through the transition region.

A general boundary layer integral prediction scheme based on existing established integral techniques for the laminar and turbulent boundary layers with an intermittency modelled transition region, has been developed and applied successfully to a range of test data.
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<td>7-10 April 1986</td>
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<td>The paper &quot;Application of a microcomputer for control, data acquisition and modelling in transitional boundary layer studies&quot; by Fraser C J, Milne J S and Gardiner, I D was presented by C J Fraser.</td>
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<td>a</td>
<td>-</td>
<td>Tani's profile parameter</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>-</td>
<td>constant in the law of the wall</td>
<td>-</td>
</tr>
<tr>
<td>Cf</td>
<td>-</td>
<td>local skin friction coefficient</td>
<td>-</td>
</tr>
<tr>
<td>Cp</td>
<td>-</td>
<td>Pressure coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$H_{12}$ or $H_{32}$</td>
<td>$\delta^*/\theta$</td>
<td>Shape factor</td>
<td>-</td>
</tr>
<tr>
<td>k</td>
<td>-</td>
<td>Von Karman constant in law of the wall = 0.41</td>
<td>-</td>
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<tr>
<td>$R_X$</td>
<td>$\frac{x U_{\infty}}{\nu}$</td>
<td>Length Reynolds Number</td>
<td>-</td>
</tr>
<tr>
<td>$R_\sigma$</td>
<td>$\sigma U_{\infty} \nu$</td>
<td>Transition length Reynolds Number</td>
<td>-</td>
</tr>
<tr>
<td>$R_\lambda$</td>
<td>$\frac{\lambda U_{\infty}}{\nu}$</td>
<td>Transition length Reynolds Number</td>
<td>-</td>
</tr>
<tr>
<td>$R_\theta$</td>
<td>$\frac{\theta U_{\infty}}{\nu}$</td>
<td>Momentum thickness Reynolds Number</td>
<td>-</td>
</tr>
<tr>
<td>$R_{\delta^*}$</td>
<td>$\frac{\delta^* U_{\infty}}{\nu}$</td>
<td>displacement thickness Reynolds Number</td>
<td>-</td>
</tr>
<tr>
<td>$Tu$</td>
<td>$\sqrt{\frac{\overline{u^2}}{U_{\infty}^2}} \times 100$</td>
<td>freestream turbulence intensity</td>
<td>%</td>
</tr>
<tr>
<td>$\overline{u}$</td>
<td>-</td>
<td>local mean velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{\infty}$</td>
<td>-</td>
<td>freestream velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_0$</td>
<td>-</td>
<td>freestream velocity at leading edge</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_\tau$</td>
<td>$\sqrt{\tau_0/\rho}$</td>
<td>Wall friction velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$u^+$</td>
<td>$\frac{\overline{u}}{U_{\tau}}$</td>
<td>dimensionless velocity</td>
<td>-</td>
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<td>-----------------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
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<tr>
<td>$u'$, $v'$, $w'$</td>
<td>fluctuating Velocity components in x, y, z directions respectively</td>
<td>m/s</td>
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<tr>
<td>$\bar{x}$</td>
<td>location of the 50% intermittency point</td>
<td>mm</td>
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</tr>
<tr>
<td>$x$</td>
<td>streamwise co-ordinate</td>
<td>mm</td>
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</tr>
<tr>
<td>$y$</td>
<td>transverse co-ordinate</td>
<td>mm</td>
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</tr>
<tr>
<td>$z$</td>
<td>spanwise co-ordinate</td>
<td>mm</td>
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<tr>
<td>$y^+$</td>
<td>dimensionless y co-ordinate</td>
<td>-</td>
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<tr>
<td>$\overline{y}$</td>
<td>mean 'near wall' intermittency</td>
<td>-</td>
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<tr>
<td>$\delta$</td>
<td>boundary layer thickness at $U = 0.995 U_\infty$</td>
<td>mm</td>
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<td>$\delta^*$</td>
<td>$\int_0^\infty \left(1 - \bar{U}/U_\infty\right) dy$</td>
<td>displacement thickness</td>
<td>mm</td>
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<tr>
<td>$\theta$</td>
<td>$\int_0^\infty U_\infty \left(1 - \bar{U}/U_\infty\right) dy$</td>
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<tr>
<td>$\delta^{**}$</td>
<td>$\int_0^\infty \bar{U}/U_\infty \left(1 - \left(U/U_\infty\right)^2\right) dy$</td>
<td>Energy thickness</td>
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<td>$\lambda$</td>
<td>transition normalising length</td>
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<tr>
<td>$\lambda_p$</td>
<td>$\frac{\delta^2 \frac{dU_\infty}{dx}}{v}$</td>
<td>Pohlhausen pressure parameter</td>
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<tr>
<td>$\lambda_\theta$</td>
<td>$\frac{\theta^2 \frac{dU_\infty}{dx}}{v}$</td>
<td>modified Pohlhausen/Thwaites parameter</td>
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<tr>
<td>$\mu$</td>
<td>fluid dynamic viscosity</td>
<td>kg/ms</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>fluid kinematic viscosity</td>
<td>m²/s</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>air density</td>
<td>kg/m³</td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>Coles &quot;wake&quot; profile parameter</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of mean intermittency distribution</td>
<td>mm</td>
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<tr>
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<td>Function</td>
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<td>( \tau_0 )</td>
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<td>( \zeta )</td>
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<td>( \zeta )</td>
<td>( \frac{x - \bar{x}}{\sigma} )</td>
<td>transition normalising co-ordinate</td>
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<tr>
<td>( \eta )</td>
<td>( \frac{x - x_s}{x_e - x_s} )</td>
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<tr>
<td>( \ell_1(\lambda_0) )</td>
<td>-</td>
<td>Thwaites relationships between ( \ell_1 ) and ( \lambda_0 )</td>
<td>-</td>
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**Subscripts**

- \( e \) - related to the end of transition
- \( i \) - denoting initial conditions
- \( l \) - relating to transition length
- \( L \) - related to the laminar region
- \( o \) - denoting conditions at the leading edge
- \( s \) - related to the start of transition
- \( t \) - related to the transition region
- \( T \) - related to the turbulent region

Other symbols, not noted here, are defined within the text.
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STATEMENT OF OBJECTIVES

1) To review the literature on the current conceptional understanding of the transition process.

2) To improve the flow in the existing boundary layer wind tunnel test facility.

3) To investigate the suitability of a microcomputer based system with analogue-to-digital conversion facilities for the acquisition of data, from a hot wire signal, in laminar, turbulent and transitional boundary layers.

4) To develop suitable microcomputer software for the control of an automatic boundary layer traverse; for the logging of velocity profile data and for the subsequent analysis and reduction.

5) To set up flows with different combinations of pressure gradient and freestream turbulence level and to measure the boundary layer development under the influence of these effects.

6) To obtain transition onset and length data in both zero and non zero pressure gradients at a range of freestream turbulence levels.

7) To investigate the concept of statistical similarity of transition regions in non-zero pressure gradients and to consider methods of representing this similarity, if it exists.

8) To review the current methods of predicting the onset and length of transition.

9) To investigate the effect of freestream turbulence level and freestream pressure distribution on the transition length and to correlate these effects.

10) To develop a general boundary layer integral prediction model, based on an intermittency weighted transition region, for the development of the transitional boundary layer growth and to develop microcomputer codes for this model.
CHAPTER 1

Introduction

1.1 Early experiments

Until Prandtl's epoch making lecture at the beginning of this century the science of fluid dynamics had been developing along two different branches; one being the dynamics of frictionless fluids called *hydrodynamics*, which was regarded as an academic subject incapable of practical application, and the other being the empirical science of *hydraulics* based on correlations of large amounts of experimental data. This diversification arose from the inability of the hydrodynamicists to predict real solutions to practical engineering problems. Prandtl, with his paper on "Fluid Motion with Very Small Friction" read before the Mathematical Congress in Heidelberg in 1904, took the first steps to unifying these two branches by showing that it was possible to analyse viscous flows precisely in cases which had great practical importance. Prandtl described, with the aid of simple experiments, how the flow around a body could be divided into two regions: A thin layer in contact with the surface in which viscous forces were significant and the remaining region outside this layer where viscous forces can be neglected. Although acceptance of Prandtl's paper was initially very slow, it is now considered to mark the birth of modern boundary layer theory.

Even before Prandtl had presented his 1904 paper and established the boundary layer equations, Osborne Reynolds (1883) had applied himself to the problem of transition. Reynolds postulated that the breakdown of a laminar flow to turbulence was
a consequence of instability in the laminar flow. This hypothesis which was further developed by Rayleigh is now known as the Reynolds-Rayleigh hypothesis and to this day is still highly regarded.

In 1914 Prandtl carried out his famous experiments on spheres and observed that the flow in a boundary layer could also be either laminar or turbulent and furthermore, that the position at which transition occurred significantly affected the flow around a body and hence the calculation of the drag on the body.

1.2 Stability of laminar flow

Stability theory for viscous fluid flows was developed independently by both Orr (1907) and Sommerfeld (1909) and resulted in what is now known as the Orr-Sommerfeld equation. This equation was derived from a finite disturbance analysis of the Navier-Stokes and continuity equations and is the starting point for all stability calculations. No practical solution to this equation was obtained until the late 1920's when, not surprisingly, the breakthrough came from one of Prandtl's students, Tollmein (1929) who computed theoretically the critical Reynolds number at which the laminar flow becomes unstable to a travelling wave type of disturbance. Schlichting (1933) later extended Tollmein's calculations to amplified two dimensional disturbances which are now recognised as Tollmein-Schlichting waves.

Despite the notable achievement of both Tollmein and Schlichting their work was disregarded for almost a decade until two of Dryden's co-workers, Schubauer and Skramstad (1943)
conducted experiments on a flat plate in a wind tunnel with very low residual turbulence. In these experiments Schubauer and Skramstad forced the boundary layer to oscillate by vibrating a thin magnetic ribbon immersed in the layer. At certain Reynolds numbers they observed that the oscillations were amplified and that transition to turbulence was preceded by these amplified oscillations. These experiments were regarded as confirmation of the previously purely theoretical concept of Tollmein-Schlichting waves and critical Reynolds number. The reason these observations had not been made in earlier experiments was considered to be due to the high levels of freestream turbulence, typical of earlier experiments, masking the existence of amplified waves.

1.3 Transition to turbulence

In 1936 Dryden observed that near the beginning of transition turbulent bursts occurred randomly and at infrequent intervals and that further downstream the bursts occurred more frequently and were of longer duration until finally the flow was continuously turbulent. The intermittent appearance of the turbulence in this region was interpreted by Dryden as a wandering irregular line of abrupt transition about a mean position. However, it is now certain that this interpretation was incorrect and that the so-called transition region is composed of Emmons (1951) type turbulent spots which grow in size as they are transported downstream.

Emmons advanced the concept of turbulent spots on the basis of experiments conducted on equipment built to demonstrate a simple water table analogy to supersonic flow. In addition to
the anticipated supersonic phenomenon, Emmons noticed the appearance of strange turbulent bursts and had the foresight to recognise this as the breakdown of the laminar flow. He observed that the transition region was filled with a random collection of these turbulent bursts or spots which appeared to grow at a constant rate and independently of each other.

From these observations Emmons deduced a source density function which described the production of turbulent spots, and showed how this could be related to the probability of the flow being turbulent at a given point, namely, the intermittency factor $\gamma$.

Following Emmons paper in 1951, the existence of turbulent spots in a boundary layer was confirmed experimentally by Mitchner (1954) and Schubauer & Klebanoff (1956). Mitchner's technique of artificially generating turbulent spots by means of an electric spark was used by Schubauer & Klebanoff to make detailed measurements of the spot growth and geometry. The shape of Schubauer & Klebanoff's artificially generated turbulent spots is shown in fig. [1.3.1] below.

More recently conditional sampling techniques have been used by Wygnanski et al (1976) and Arnal (1977) to measure the mean
velocity profiles in and out of turbulent spots, in a transition region, and have shown that the flow within a turbulent spot is characteristic of a turbulent boundary layer. Gad-el-Hak et al (1981) used a rather novel flow visualisation technique to obtain an excellent series of colour photographs showing the growth of a turbulent spot on a flat plate towed through a tank of water. These photographs show that the characteristic shape of the turbulent spot remains unchanged as the spot grows and is swept downstream with the mean flow.

So far the mechanism of the process leading to turbulent flow has been elucidated on the basis of controlled experiments such as those by Klebanoff, Tidstorm & Sargent (1962) and although no theory exists for the prediction of transition, the breakdown process is qualitatively well defined.

The breakdown process begins with the amplification of Tollmein-Schlichting waves which become associated at some stage with a concentration of vorticity along discrete lines. These subsequently distort into vortex loops which themselves go through a process of distortion and extension until they finally break into localised bursts of turbulence ie turbulent spots. The turbulent spots then grow, laterally as well as axially, until they eventually coalesce to form a completely turbulent flow field.

This process can be simplified into three stages.

(i) Amplification of small disturbances.

(ii) Generation of localised areas, or spots, of turbulence.

(iii) Growth and spread of turbulent spots.
While theories of the Tollmein-Schlichting type have achieved a fair amount of success in predicting the influence of various effects on the limit of laminar stability (stage (i)), they give no indication of the point at which transition occurs (stage (ii)).

1.4 Practical significance of transition

Transition from laminar to turbulent flow is not only an important problem of fundamental research in fluid mechanics but possesses many important ramifications. For example, the drag of a body placed in a stream as well as the rate at which heat is transferred from a solid wall to a fluid moving past it, depend very strongly on whether the flow in the boundary layer is laminar or turbulent. The occurrence of transition can sometimes be beneficial, for example in delaying separation or in promoting more rapid diffusion of heat and sometimes it can be detrimental in increasing skin friction and promoting undesirable high rates of heat transfer. Whether beneficial or detrimental the accurate prediction of its position on a body is obviously of paramount importance to the computation of the boundary layer development over a body, and hence the calculation of the aerodynamic and thermodynamic performance of the body.

One rather crude method of calculating the boundary layer development on a surface is to assume that transition from the laminar to turbulent flow state occurs instantaneously at the transition point, and to overlap the laminar and turbulent boundary layer parameters at this point. This method may be substantiated in some cases when the length over which the
boundary layer degenerates from the laminar to turbulent flow state, i.e., the transition length, is small in comparison to the length of the body itself. However, in situations where the transition region occupies a significant proportion of the body surface then the length over which the transition metamorphosis takes place will be of great significance to the development of the boundary layer.

One practical example of a situation where the transition region occupies a high proportion of a body surface is a modern gas turbine blade. Turner (1971) observed that the boundary layer over a turbine blade can be transitional for up to 70% of its chord. In such a case the quality of the boundary layer prediction through the transition region can influence the blade aerodynamic efficiency and, through its impact on cooling design, the cycle efficiency and hardware durability of the turbine. Therefore accurate prediction of the boundary layer development through transition which is dependent on accurate prediction of the onset and length of transition, is of prime importance.

1.5 Prediction of transition onset

The transition point, which lies some distance downstream of the point of laminar instability, can be defined as the point at which the mean laminar boundary layer parameters begin to deviate from their typical laminar values and is normally considered to be the point where the laminar flow breaks down to random turbulence, i.e., the appearance of first turbulent spots. In general transition is known to be influenced by a number of factors such as: surface roughness, freestream turbulence,
pressure gradient, Mach number, surface curvature, Reynolds number etc. Because of the complex manner in which the various factors influence the position of the transition point and the extent of the transition region, theorists have been unable to solve the transition problem analytically. For this reason the design engineer has had to rely on empirical and semi-empirical models, based on experimental data, to obtain solutions to practical engineering problems. Obviously the accuracy of any solution will depend on the quality of the experimental data, the degree of correlation and the number of influencing factors that are accounted for in the model. Two of the most dominant factors which influence transition are the pressure gradient and freestream turbulence intensity, consequently any empirical model should, at least, account for their effects.

Various methods for predicting the position of the transition point are available, all operating on an empirical or semi-empirical basis but with varying transition criteria and basic assumptions. These methods have been reviewed by a number of researchers eg Tani (1969), Hall & Gibbings (1972), Reshotko (1976) and more recently in an excellently complete review by Arnal (1984). It is not intended therefore, to repeat this work here but merely to briefly describe the empirical and semi-empirical approach to the prediction of transition point.

Semi-empirical approach: This approach includes the "so-called" e\(^n\) methods formed on the basis of the linear stability theory and correlations from low turbulence wind tunnel experiments. The first of these methods was developed by Smith & Gamberoni (1956) and independently by Van Ingen (1956). They observed that the maximum amplification ratio of the initial disturbances, as
computed by the stability theory, at the observed position of
transition was roughly equal for all cases investigated.
According to Smith & Gamberoni, this critical value of
amplification ratio is approximately \( e^9 \).

Since this method was first introduced various modifications
to the original calculation method have been made, for example
by Jaffe, Okamura & Smith (1970) and others. However the method
remains essentially as originally developed, the key to success
of the method lying in the judicious choice of the exponent
factor, ranging anywhere from about 8 to 11.

\( e^n \) methods are only applicable for flows with low freestream
turbulence levels (say <0.2%). At higher freestream turbulence
levels the Tollmein-Schlichting mode to transition is thought to be
by-passed, transition then being due to pressure fluctuations in
the freestream, Taylor (1936).

Another method which can be classed as having a semi-empirical
approach, as it includes some theoretical elements, is that of
Van Driest & Blumer (1963). This was developed on Liepmann's (1936)
idea that transition occurred when the ratio of local turbulent to
viscous shear stress reached some critical value. By using
Taylors' (1936) hypothesis for freestream turbulence effects and
the Pohlhausen (1921) forth-degree velocity profile an expression
involving two adjustable constants has been derived for transition
Reynolds number in terms of pressure gradient parameter, \( \lambda_p \), and
freestream turbulence. The constants in this expression being
adjusted to fit experimental data.
**Wholly empirical approach** These methods are based on the assumption that the local transition Reynolds number can be determined, through correlation of controlled experimental results, as a function of the factors which influence transition. Whether the prediction based on local Reynolds number is accurate or not depends on whether all the important parameters are taken into account.

The method of Michel (1951) which comprises only one relationship between the momentum thickness Reynolds number and the axial length Reynolds number at transition, is probably the simplest of those methods. Michel's curve can be fitted by the expression.

\[ \frac{\theta_s}{\theta_s} = 1.535 R_{x_s}^{0.44} \]  

When the corresponding flow Reynolds numbers coincide with Michel's curve ie equation 1.1 then transition is "predicted". Other early methods worthy of note are those of Granville (1953) and Crabtree (1957) both using a single curve of \( R_\theta \) against a pressure gradient parameter \( \lambda_\theta \) as a transition criteria. However, Granville's method differs from Crabtree's in that he attempted to make an allowance for the upstream flow history by assuming that only the boundary layer growth after the point of stability was of any consequence. Granville therefore plotted \( (R_\theta - R_{\theta cr}) \), where \( R_{\theta cr} \) is the value of \( R_\theta \) at the stability limit, against the mean value of \( \lambda_\theta \) over the unstable part of the boundary layer. As was the case for the early methods these early methods are really only applicable to low freestream turbulence flows.
The more recent methods of Dunham (1972), Seyb (1972) and Abu-Ghannam & Shaw (1980) directly correlate the momentum thickness Reynolds number at transition against the local pressure gradient parameter $\lambda_{0g}$ and the freestream turbulence level. The most recent of these ie Abu-Ghannam and Shaw's is probably the most reliable as it is based on a vast amount of experimental data obtained from a variety of sources.

1.6 Boundary layer development through transitions (present investigations)

Assuming the transition point is known the problem then is to compute the boundary layer development through the transition region itself, the extent of which may be longer than the laminar layer which precedes it. An important parameter characterising the transitional boundary layer is the mean "near wall" intermittency factor $\overline{\gamma}$ which represents the fraction of time that the flow is turbulent. The model presented in this thesis is based on the 'so-called' "intermittency method", in which the laminar and turbulent boundary layer quantities are weighted by $\overline{\gamma}$. Thus the first task is to describe the streamwise evolution of the intermittency factor $\overline{\gamma}$ through the transition region.

There are other methods, such as that of McDonald & Fish (1973) which do not require the knowledge of $\overline{\gamma}$. Such methods involve finite difference solution of the mean flow and some form of eddy viscosity. However these methods require the use of refined finite difference grids with perhaps more than 100 grid points across the boundary layer. This makes such approaches uncomfortably slow in engineering design applications, See Forrest (1977). Such methods have not been considered further in this investigation.
Schubauer & Klebanoff (1956) measured the streamwise distribution of $\bar{\gamma}$ for a number of zero pressure gradient flows where conditions leading to transition were varied and, although in each case the transition lengths were different, the distribution followed the general shape of the Gaussian integral curve. The standard deviation $\sigma$ was calculated for each experiment, and all the data collapsed on to a single curve when $\bar{\gamma}$ was plotted as a function of the normal stream co-ordinate $\xi = \frac{x - \bar{x}}{\sigma}$ where $\bar{x} = x(\gamma=0.5)$. (The value of $\sigma$ is a measure of the spread of the data about the 50% intermittency point and, if the transition region is defined in the limits $0.01 < \gamma < 0.99$ then $\sigma$ can be related directly to the transition length).

Schubauer and Klebanoff, from these observations, postulated that transition regions in all zero pressure gradient flows, long or short, were statistically similar. This concept was corroborated by Dhawan & Narasimha (1958) although in contrast to Schubauer & Klebanoff they proposed a different distribution function of intermittency:

$$\bar{\gamma} = 1 - \exp^{-0.412\xi} \quad \text{......... 1.2}$$

where $\xi = \frac{x - x_S}{\lambda}$ is the normalised stream co-ordinate with $\lambda$ as a measure of the intermittency spread given by

$$\lambda = (x \text{ at } \bar{\gamma} = 0.75) - (x \text{ at } \bar{\gamma} = 0.25)$$

(By defining the transition region in the same limits as before $\lambda$ can also be related directly to the transition length).

A similar method with yet a different intermittency distribution function has also been proposed by Abu-Ghannam & Shaw (1980).
All three of the methods for defining the intermittency distribution have been considered in this investigation and the concept of statistical similarity for non-zero pressure gradient cases has been examined.

Unless the length of the transition region (which can be related directly to $\sigma$ or $\lambda$) is known, none of the methods constitute a means of calculating the streamwise intermittency distribution $\gamma$. For this reason Dhawan & Narasimha proposed the existence of a relationship between the transition Reynolds number ($R_{X_S}$) and a transition length Reynolds number based on $\lambda$ i.e ($R\lambda$):

$$R\lambda = R_{X_S}^{0.8} \quad \ldots \ldots \ldots \quad 1.2$$

This relationship, although known to be in error by more than 100% in some cases and agreed to be very approximate by Dhawan & Narasimha in their original paper, is used as the basis of many prediction methods which require the transition length to be known e.g. Abu-Ghannam & Shaw (1980), Brown & Burton (1978), Fraser (1979), Martin et al (1978).

The validity of this relationship is reviewed and a new correlation, based on data gathered during this investigation and on the limited amount of existing data available, is proposed for defining the transition length. The new correlation accounts directly for the effect of freestream turbulence and pressure gradient on the transition length and is in the form:

$$R_\sigma = f(Tu, \lambda_\sigma) \quad \ldots \ldots \ldots \quad 1.4$$

The development of this relationship is discussed in detail in Chapter 6.
Dhawan & Narasimha also proposed that the transition region could adequately be described as a region of alternate laminar and turbulent flow. With the intermittency distribution known they assumed that the transitional mean velocity profiles could be expressed as an intermittency weighted average of the laminar and turbulent velocity profiles ie:

$$
\left( \frac{u}{U_\infty} \right)_t = (1 - \gamma) \left( \frac{u}{U_\infty} \right)_L + \gamma \left( \frac{u}{U_\infty} \right)_T
$$

Qualitative measurements made in the transition region in the present investigation substantiate this model, as do the detailed conditionally sampled measurements of Arnal (1977) and Wygnanski (1976).

These observations of Dhawan & Narasimha along with the intermittency distribution of Schubauer & Klebanoff and the present correlation for transition length are formulated into a computational model for predicting the boundary layer development through transition. The laminar and turbulent boundary layer components are obtained from the established integral techniques of Tani (1954) for the laminar boundary layer and Alber (1968) for the turbulent boundary layer.

The development of the model is discussed in detail in Chapter 7 and a comparison of predictions obtained from the model against a sample of past and present data is made.
1.7 Microcomputer involvement

During the last decade the most significant improvements in instrumentation and measurement have been centred on the development of microelectronics, with particular reference to microprocessors which have added a new dimension of intelligence and control in measurement systems. The operational flexibility of the microcomputer allows the same machine a functional role in the data taking process, the analysis and reduction of the primary data and the mathematical modelling of the observed phenomenon.

A large proportion of the present study was devoted to the development and commissioning of a microcomputer data acquisition and control system based on a BBC microcomputer with a Double Disc drive unit for the storage of software and data files. This system contributed significantly to the speed at which reliable accurate data could be obtained and processed. The computational model described in the previous section was also programmed to run on the same BBC micro thus exploiting the full potential of the system.

The development of this system has resulted in the publication of two papers i.e Milne, Fraser & Gardiner (1985) and Fraser, Milne & Gardiner (1986). A further paper by Fraser, Gardiner & Milne (1987) is to be presented at the 5th International Conference on "Numerical Methods in Laminar and Turbulent Flow" to be held in Montreal, CANADA in July 6th - July 10th 1987.
CHAPTER TWO

EXPERIMENTAL FACILITIES

2.1 Wind tunnel test facility

All experiments during this investigation were conducted in a purpose built, open return, boundary layer wind tunnel. Details of the design of this tunnel are given by Fraser (1979). The tunnel was originally designed for the study of two dimensional, incompressible flat plate boundary layer flow and has an adjustable roof which enables the boundary layer to be subjected to adverse, zero and favourable pressure gradients. Moderately low freestream turbulence levels (around 0.35%) can be obtained in this facility and it has recently been modified to allow higher turbulence levels to be generated within the test section through the use of various turbulence generating grids. A schematic diagram of the tunnel is shown in fig. 2.1.1.

The tunnel consists of a series of damping screens, an inlet contraction, a rectangular working section, a square to round section diffuser and a variable speed 2 hp D.C. motor which drives a six blade propeller fan.

The damping screens, situated upstream of the inlet contraction, are designed for the double purpose of reducing the spanwise nonuniformity in the flow, as suggested by East (1972), and reducing the freestream turbulence level by removing large scale eddies and inducing lower scale eddies which rapidly decay downstream of the grids, Dryden & Schubauer (1947). The inlet contraction which is of rectangular section, has an aspect ratio of 2/1 and an area reduction ratio of 9/1. The contraction is designed to further reduce the freestream turbulence and accelerate the flow.
into the working section. Downstream of the inlet contraction is the working section. This is of rectangular cross section 227 mm x 450 mm x 2.5 m with an adjustable height roof to enable variable pressure gradient flows to be set up in the test section.

Situated within the tunnel working section is the instrument carriage which was designed to give three-dimensional flexibility for the hot wire sensor positioning. The carriage runs on two horizontal rails fixed to the tunnel side walls, which allows streamwise flexibility in the probe positioning, and a cross slide, to which all the necessary measuring equipment and vertical traversing gear can be attached, allows for spanwise positioning. Positioning in the spanwise and streamwise direction is done manually from inside the working section with the vertical traversing being carried out remotely using the DISA Sweep drive unit (type 52B01) in conjunction with a stepper motor (type 52C01) which drives through reduction gearing, a rack and pinnion. The rack being ultimately attached to the probe sensing head. A photograph of the probe traversing mechanism is shown in fig. [2.1.2].

A pitotstatic tube, coupled to an inclined manometer, is in permanent place at the entrance to the working section above the plate leading edge. This enables the reference approach velocity at the leading edge of the plate to be continuously monitored. Access to the working section is via four hinged doors on the front wall.

A flexible coupling joins the exit of the working section to the diffuser. The purpose of this flexible coupling was twofold, firstly, to prevent vibrations from the fan and motor being transmitted to the working section and secondly to provide a pliable seal between the variable height roof and the diffuser.
The diffuser merges from a square to a round section over its 1.5 m length. The section at the upstream end is 450 mm x 450 mm and the diameter at the downstream end is 800 mm.

The six blade fan propeller is housed in a 700 mm long cylindrical casing and is driven by a 2 hp variable speed motor. The motor has a maximum speed of 1440 rpm giving a maximum reference velocity at the entrance to the working section of nominally 20 m/s.

### 2.2 The boundary layer Plate

The boundary layer plate is a 6 mm thick aluminium sheet 2.4 m long and completely spans the working section. The plate is fixed to two rails which are bolted through the tunnel floor onto the main supporting framework. Along the centre line of the plate are a series of pressure tappings set at 50 mm pitch and these tappings are connected to a multitube inclined manometer. Originally the plate was positioned 50 mm above the working section floor at zero incidence to the approach flow and the leading edge was symmetrically sharpened and bent downwards to ensure that the stagnation point would occur on the upper surface of the plate leading edge. Previous results from earlier work, Fraser (1979), showed that natural transition on the plate occurred at values of Rx well below those obtained by other researchers such as Van Driest & Blumer (1963), Hall & Gibbings (1972) and Abu-Ghannam & Shaw (1980). This early transition was initially thought to be caused by the leading edge geometry so, the front of the plate was removed and a new straight symmetrically shaped leading edge was machined on the plate and hand worked to merge tangentially to the plate.
horizontal surface. To ensure the stagnation point would occur on the top surface of the leading edge the whole plate was then inclined at $-1^\circ$ to the oncoming flow. Fig. [2.2.1] shows the new leading edge geometry. To obtain this $-1^\circ$ of incidence the plate leading edge was lowered to 23 mm from the tunnel floor rather than the trailing edge being raised. However, as can be seen from the tunnel approach velocity profiles fig. [2.2.2] the leading edge of the plate is still well clear of the boundary layer developing on the tunnel floor.

2.3 Preliminary Tests & Tunnel Modifications

An initial study to determine the flow regimes over the modified flat plate, in a zero pressure gradient, was carried out by positioning the probe approximately 1 mm from the plate surface and observing the trace from the constant temperature anemometer (DISA 55M10), on an oscilloscope, at numerous spanwise and streamwise positions. This study gave an indication of the regions of laminar, transitional and turbulent flow over the plate. As can be seen from fig. [2.3.1], there appears to be large disturbances which emanate from the tunnel side walls and grow downstream, progressively encroaching into the flat plate test flow. This phenomenon, although not often reported, is thought to be a common occurrence in boundary layer wind tunnels. It was observed by Coles & Savas (1979) who stated, "The useful region of the plate surface was severely limited by transverse contamination from the sidewalls", and more recently by Blair (1982). For this reason all test measurements were restricted to the tunnel centreline.
Even with this restriction imposed, values of \( R_{x_s} \) still fell far short of those expected therefore, it was decided to make further improvements to the tunnel to, at least, delay the start of the transition to obtain values of \( R_{x_s} \) approaching those of Abu-Ghannam & Shaw (1980).

Initial improvements were

(i) A suction port was added to the underside of the tunnel 500 mm from the leading edge in an attempt to improve the flow over the leading edge of the plate. Oil and smoke flow visualisation techniques showed that this suction made no difference to the flow over the leading edge and in fact the flow in this region was fairly good with no signs of separation occurring on the topside near the leading edge.

(ii) The tunnel roof side wall seals were replaced as smoke tests revealed an inflow to the tunnel working section from the atmosphere, through inadequate sealing at the joint between the adjustable roof and the tunnel side walls.

(iii) The seals around the working section access doors were replaced as the original "draftproofing" had perished.

None of these improvements delayed the start of transition on the plate, in fact it was discovered after all these "improvements" had been made that transition was actually occurring earlier than ever on the test surface.
This was very disappointing, but after much deliberation on this problem the reason for the early transition was eventually traced to the fact that the three turbulence damping screens at the intake to the tunnel had been cleaned, removing a fair quantity of dust which had accumulated on them and appeared to be increasing their effectiveness. The addition of two further screens, one of a manmade micromesh fabric (used for wind breaks) was placed at the front of the bank of screens and the other, a 40 mesh stainless-steel wire mesh grid, placed at the rear of the bank of screens. Details of the screens are given in fig. [2.3.2]. The addition of these extra screens did not significantly decrease the level of natural freestream turbulence in the tunnel but did greatly improve the flow over the plate and established values of $R_{x_s}$, on the centre line of the plate, of the same order of those obtained by Abu-Ghannam & Shaw (1980), fig. [2.3.3].

2.4 Turbulence Generating Grids

The various freestream turbulence levels required throughout this investigation were produced by placing turbulence generating grids close to the contraction entrance, about 400 mm downstream of the contraction front edge, see fig. [2.1.1]. This arrangement differs from that used in many of the early investigations of this subject in that the grids are located in the inlet contraction and not downstream of it at the entrance to the test section. The benefits derived from this, as reported by Blair (1983), are that the turbulence generated in the test section is more homogeneous and has a much lower decay rate along the test section. This is illustrated in fig. [2.4.1].
The advantage of locating the grids in this position requires that coarser grids be used to achieve given test section turbulence levels.

The grids designed gave turbulence intensities of approximately 0.45%, 0.75% and 1.45% in the test section of the tunnel. These grids will now be referred to as grid 1, grid 2 and grid 3 respectively.

Grid 1: is a wire grid of mesh size 25 mm and rod diameter 2.5 mm.

Grid 2: is a wooden grid of mesh size 25 mm and rod diameter 5.5 mm.

Grid 3: is a wooden grid of mesh size 38 mm and is made from 6 x 12 mm rectangular section strips.

Further details of the grids are given in fig. [2.4.2].

2.5 Freestream Pressure Gradients

The range of pressure gradients required for this investigation were introduced by adjusting the variable height roof to give the required velocity distribution within the test section. This proved to be a difficult and very time consuming task as slight alterations in the roof height could affect the entire velocity distribution over the plate.

The procedure adopted in setting up the pressure distributions was to initially adjust the roof to give, crudely, the required static pressure distribution along the plate, measured from the plate static tappings via a multitube manometer. Fine adjustment of the roof was then implemented by measuring the freestream velocity distribution, with a hot wire, and adjusting the roof accordingly.

Four different roof settings were used to illustrate the effects of favourable, zero and adverse pressure gradients. All
the roof settings gave reasonably linear velocity distributions over the test length of the plate except for the favourable gradient setting which, due to the tunnel geometrical constraints, was non-linear in the region of the leading edge.

The pressure distributions expressed in terms of the pressure coefficient, $C_p$, are shown in fig. [2.5.1] along with the corresponding velocity distributions.

2.6 Hot Wire Instrumentation

DISA hot wire instrumentation was used consistently throughout the duration of this project. Miniature boundary layer probes (55P15) were connected via a probe support (55H21) and a 5 m length of coaxial cable to the (55M01) Main Unit fitted with a (55M10) Bridge operating in the constant temperature mode. A simplified schematic diagram of the constant temperature anemometer is shown in fig. [2.6.1].

In essence, the constant temperature anemometer consists of a Wheatstone bridge, with the probe wire serving as one of the bridge arms, and a servo amplifier. The bridge is in balance if the probe resistance and the adjacent bridge resistance $R_v$ (fig. [2.6.1]) are equal, so a voltage applied to the top of the bridge will produce no out of balance, or error voltage across the bridge. Any flow over the probe will have the effect of cooling the wire, resulting in a small change in probe resistance which in turn produces an error voltage across the bridge. This is amplified in the servo amplifier and fed back to the bridge top, causing the bridge current to increase and the probe temperature to eventually return to its original value. The voltage which is fed to the bridge top to maintain the probe
temperature can be related to the fluid velocity by calibration. The response of the system is optimised by subjecting the probe to a square wave input and adjusting the bridge gain and upper operating cut off frequency. Fine tuning is achieved by adjustment of the Q and L cable compensation potentiometers.

The voltage output from the constant temperature anemometer is non-linearly related to the fluid velocity over the probe. In order to obtain a linear relationship, the signal from anemometer is passed through a 55M25 Lineariser, which is basically an analogue computer that linearises the anemometer signal by means of a transfer function composed of exponential and square root terms. A pictorial representation of the non-linearised and linearised hot wire signal is shown in fig. [2.6.2] and an actual linearised calibration is shown in fig. [2.6.3].

A spectral analysis of the turbulence signal from a typical turbulent boundary layer shows that the turbulent energy is contained below a frequency of approximately 2 kHz, therefore the signal output from the lineariser is passed through the auxiliary unit (55D25) and filtered at a -3dB cut off frequency of 2 kHz.

The signal from the auxiliary unit is then fed to a Digital volt meter (55D30) for measurement of mean velocity and an r.m.s voltmeter (55D35) for measurement of the r.m.s. value of the velocity fluctuation.

The vertical positioning of the probe was carried out remotely using the DISA sweep drive unit (52B01) in conjunction with a stepper motor (52C01) and traverse mechanism (55H01). A photograph of the instrumentation bank is given in fig. [2.6.4] and a schematic layout is shown in fig. [2.6.5].
2.7 Probe Linearisation

As indicated in the previous section, linearisation of the hot wire probe was achieved by means of the DISA (55M25) lineariser. This is a complex piece of apparatus with a fairly comprehensive set-up procedure. In order to simplify this linearisation procedure a computer program, for a BBC microcomputer, was developed that enabled a graphical output of the linearisation to be viewed on a monitor during the set-up procedure. This enabled new probes to be fairly quickly and accurately linearised. Details of this program and the set-up procedure are given in Appendix 2.

The probes were linearised such that the hot wire output voltage corresponded to 1/10th of the fluid velocity. The freestream velocity, measured by the Hot wire, was checked against a pitotstatic reading before and after each traverse and if the hot wire velocity was in error by more than 2% the profile was rejected and the probe was recalibrated. Recalibration of a probe already in use involved measuring a set of velocities in the test range, against a pitotstatic and usually only minor adjustments to the "Gain High" and "Exponent Factor" settings on the lineariser was all that was required. It was found, however, that after a period of time, the stability of the probes deteriorated to a point where they developed such a significant drift in their calibration that they became unusable.

2.8 Intermittency Measurement

From the outset of the project it was obvious that one of the most important parameters to be measured was that of intermittency in the transition region. Intermittency was first
measured by Townsend (1948) who used the term $\gamma$ as the intermittency factor and defined it as the fraction of time a given signal is turbulent. For $\gamma = 0$ the flow is laminar all the time and for $\gamma = 1$ the flow is turbulent all the time.

Intermittency is observed in two quite different situations i.e., in the breakdown of a laminar shear flow to turbulence, a process which normally occurs over an appreciable streamwise distance, and at the freestream interface of a fully turbulent shear flow where the interface of the turbulence fluctuates with time so that over an appreciable cross-stream distance, the flow alternates between turbulent and substantially irrotational motion. It is the former of these two situations which the present investigation is primarily concerned.

There are various methods by which the intermittency factor can be measured. One of the first methods used by Townsend (1948), Klebanoff (1955) and Sandborn (1959) was that of the flatness factor. The flatness for $u'$ is given as:

$$\text{flatness factor} = \frac{\overline{u'^4}}{(\overline{u'^2})^2} \quad \ldots \quad 2.1$$

As the probability distribution of the interface between the turbulent and non-turbulent fluid is approximately Gaussian, then near the wall, where the intermittency is unity, the flatness factor corresponds closely to the Gaussian value of 3.0. By considering the intermittency as an on/off process the value of $\gamma$ can then be found from

$$\gamma = \frac{3}{\overline{u'^4} / (\overline{u'^2})^2} \quad \ldots \quad 2.2$$
Other methods developed along the lines of Corrsin & Kistler (1954), which are popular with more recent researchers, Sharma, Wells et al (1982), Murlis et al (1982), are usually termed on/off Velocity-intermittency methods. The basis of these methods is to modify the hot wire signal to enable distinct discrimination to be made between laminar and turbulent flow regimes. A schematic diagram of this process is shown in fig. [2.8.1]

Fiedler & Head (1966) have had some success in measuring the intermittency through a turbulent boundary layer using a photo-cell instead of a hot wire anemometer to obtain the basic signal. Smoke is introduced into the boundary layer and illuminated by a light normal to the surface making a cross-section of the boundary layer visible. The relative illumination of the boundary layer and free-stream are then detected by a photo cell and the output from this photo cell is passed through the same intermittency measuring circuitry as used for the hot wire signal shown in fig.[2.8.1 (b)].

More recently Murlis et al (1982) have developed a temperature-intermittency scheme using a cold-wire and a heated plate. The advantage of this is that the temperature in a heated flow is larger than that in the freestream everywhere within the turbulence, unlike velocity-intermittency schemes where the discriminating fluctuating velocity of the turbulence can be negative as well as positive and even the square of the fluctuating velocity component will have occasional zeros.

The latter two methods mentioned above have been developed for measurement of the intermittency distribution through a turbulent boundary layer and it is doubtful if they would be of
any use when making intermittency measurements in a region of breakdown from laminar to turbulent flow.

For this reason, and the fact that hot wire instrumentation and a DISA APA system were readily available, an on/off velocity-intermittency system was developed for the measurement of intermittency for this investigation. A circuit diagram of the hot wire signal modifier is shown in fig. [2.8.1 (a)].

The signal modifier consists of 3 parts:

(i) Removal of the D.C. component of the hot wire signal leaving only the time dependent velocity signal.

(ii) Amplification and full wave rectification of the signal.

(iii) Removal of the zeros and smoothing to give an approximate square wave.

The signal from the signal modifier is then passed to the DISA comparitor (52B10) which is fed with a triggering level. The comparitor produces 5v time dependent pulses corresponding to the approximate square pulses produced by the signal modifier as shown in fig. [2.8.2 (a)]. This signal is then passed to an averaging D.V.M. which gives a reading of 5v for \( \gamma = 1 \) ie all the modified signal is above the triggering level, and a reading of 0v for \( \gamma = 0 \), ie all the modified signal is below the triggering level. Values in between 0v and 5v correspond to intermittency values between 0 and 1.

In practice the triggering level was set for each flow by visual observation, on a dual beam oscilloscope, of simultaneous traces of the modified hot wire signal and corresponding triggered signal from the DISA comparitor.

Arnal (1984) noticed that in high freestream turbulence and adverse pressure gradient flows the intermittency is less easily
discriminated. This is because of high amplitude, but low frequency disturbances that are present in the laminar portion of the flow making the choice of an appropriate detection signal unclear. This problem was overcome by passing the "raw" hot wire signal through a 50-100 Hz HP filter, depending on the flow, before passing it to the signal modifier.

This effectively filters out the low frequency signal leaving prominent turbulent bursts which can easily be discriminated from the surrounding laminar flow as shown in fig. [2.8.2 (b)]. Fig. [2.8.2 (c)] shows a comparison between the filtered hot wire signal and the modified signal and as can be seen from this figure the "approximate" square wave pulses from the signal modifier correspond to the turbulent bursts from the filtered signal.

2.9 Measurements of Cf using a Preston tube

The Preston tube is essentially a circular total head and static tube pair, details of which are given in fig. [2.9.1]. The differential pressure measured between the two tubes can then be converted to a wall shear stress and skin friction coefficient using the calibration of Patel (1965).

\[ \text{ie } y^* = 0.8287 - 0.1381x^* + 0.1437x^{*2} - 0.0060x^{*3} \quad \ldots \quad 2.3 \]

for 1.5 < \(y^*\) < 3.5

or \(y^* = 0.5x^* + 0.037 \quad \ldots \quad 2.4\)

for \(y^* < 1.5\)

where \(x^* = \log_{10} \frac{\Delta P_d^2}{4\rho v^2}\) and \(y^* = \log_{10} \frac{T_d^2}{4\rho v^2}\)

\(\Delta P_d\) - Preston tube pressure differential

\(d\) - Preston tube external diameter
The local skin friction coefficient can then be calculated from:

\[ C_f = \frac{2 \frac{1}{2} \rho \nu}{p U^2} \]

……….. 2.5

Details of the Preston tubes used are given by Fraser (1979). Only a limited number of measurements using the Preston tube were made throughout the duration of the experimental investigation. The values of skin friction coefficient obtained from these measurements were mostly used as an independent check on the values obtained directly from the universal turbulent boundary layer profile and from the correlations of Ludwig and Tillman (1950) and White (1974). Details of these are given in Chapter 4.
Fig. 2.1.1 Schematic layout of Boundary Layer Wind Tunnel
Fig. 2.1.2  **Probe Traversing Mechanism**

Fig. 2.2.1  **Leading edge geometry**
Velocity profiles measured at 30 mm upstream of plate Leading Edge

Fig. 2.2.2 Leading Edge Approach Velocity profiles
Plan View of flat plate

Fig. 2.3.1 Tunnel side wall disturbances
<table>
<thead>
<tr>
<th>Screen No.</th>
<th>Mesh</th>
<th>Wire Ø (mm)</th>
<th>Screen Re No. (U&lt;sub&gt;SC&lt;/sub&gt; = 2m/s)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>(NYLON MICRO-MESH MATERIAL)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20x20</td>
<td>0.25</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>20x20</td>
<td>0.25</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>30x30</td>
<td>0.20</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>40x40</td>
<td>0.15</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 2.3.2 Turbulence Damping Screen Details
Fig. 2.3.3 Graph of $R_{x_s}$ against $Tu\%$ for Zero Pressure Gradient Flows
Fig. 2.4.1 Turbulence distributions along plate

- No Grid
- Grid 1
+ Grid 2
○ Grid 3

Adverse

measurement apparatus intruding into boundary layer

Favourable
<table>
<thead>
<tr>
<th>GRID No.</th>
<th>b</th>
<th>M</th>
<th>t</th>
<th>$\frac{M}{b}$</th>
<th>% Open Area</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>2.5</td>
<td>25</td>
<td>-</td>
<td>10</td>
<td>82</td>
</tr>
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<td>38</td>
<td>6.25</td>
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</tr>
</tbody>
</table>

Fig. 2.4.2 Details of Turbulence Generating Grids
Fig. 2.5.1 - Details of pressure Gradients

- X - 1st Adverse
- O - 2nd Adverse
- • - Zero
- ▲ - Favourable

U₀ = 18.1 m/s
Fig. 2.6.1 Schematic diagram of Constant Temperature anemometer
Fig. 2.6.2 Pictorial representation of linearised and non-linearised hot wire probes
Fig. 2.6.3  Hot Wire Calibration Curve
Fig. 2.6.4. *Hot Wire Apparatus*
Fig. 2.6.5 Schematic layout of Hot Wire Apparatus
Input from Hot Wire

![Circuit diagram of signal modifier](image)

Fig. 2.8.1 (a) Circuit diagram of signal modifier

- Input Hot Wire Signal
- H.P. Filter
- Removal of D.C. Component
- Amplification & Rectification
- Removal of zeros & smooth
- DISA comparator
- D.C. Triggering level

Output to D.V.M.

Fig. 2.8.1 (b) Schematic layout of intermittency meter with typical signal outputs

- Trigger level
- 5v
- 0v
Trigger level

signal from signal modifier

signal from DISA comparator

Fig. 2.82 (a) Comparison between modified signal and signal from DISA comparator

filtered signal (100 Hz H.P.)

unfiltered signal

Fig. 2.82 (b) Comparison between filtered and unfiltered Hot wire signals
Fig. 2.8.2 (c) Comparison between signal from the intermittency signal modifier and the filtered Hot wire signal
Fig. 2.9.1 Details of Preston Tube

Static Tube O.D. = 1.41 mm
Total Head Tube O.D. = 1.41 mm
3.1 Introduction

The microcomputer has, over the last decade, become an important element in measurement and control systems although, in the early stages of their development, microcomputers were regarded as nothing more than toys by "mainframe and mini" users. Their recent increase in stature has come as a result of improved speed and memory capability enabling the traditional engineering data logging and processing systems to be replaced by much more versatile microcomputer systems. The prime benefits of microcomputer based systems are that they can perform tests quickly with good repeatability and, as well as collecting data, can process this data with very little delay. The processed results can then be displayed using the inherent powerful graphics usually associated with good microcomputers, and a hard copy of the display can be obtained direct from the printer using a "screen dump" program or from a graph plotter connected to the microcomputer. A further advantage of microcomputer control and data acquisition systems is that alterations can normally be made easily by changing software to modify parameters rather than physically changing hardware.

The design and development of a microcomputer control and data acquisition system for application to experiments on transitional boundary layers, including the signal conditioning instrumentation and the computer hardware and software, is described in the following sections.
3.2 Transmission of Data

One of the first considerations in selecting a microcomputer to be used for a data acquisition system is how the data is to be acquired by the computer. There are various methods of passing information from instruments such as a DVM or signal generator to the microcomputer; one method is via a common digital transmission standard, for example the IEEE-488 bus which was developed to allow standardised interconnection of the increasing number of intelligent instruments used in laboratories. With this transmission standard the microcomputer becomes the controller capable of monitoring devices termed listeners which receive data over the IEEE-488 bus and talkers which transmit data on the bus. Another standard bus is the serial type RS232 which is normally associated with peripheral equipment such as VDUs and printers.

If the instrumentation, or microcomputer, is not equipped with an interface to allow connection to one of these standard bus structures but the instruments have analogue outputs related to the physical quantities being measured, then a cheaper and probably more common method of passing information to the microcomputer is via an analogue-to-digital converter or ADC. This device, as the name suggests, converts an analogue signal to a digital signal that can then be processed by the microcomputer. Many microcomputers now have built in on board ADCs but those that do not, usually have the facility to enable peripheral devices such as ADCs to be added on by direct connection to the machine's bus structure.
The DISA 5600 series hot wire anemometer equipment has an interface to allow digital information to be passed over the IEEE-488 bus but the 55M series described in Chapter 2, and used in this investigation has no such facility. It does however output voltages which are representative of the quantities being measured enabling measurements from this equipment to be passed to the computer via an analogue-to-digital converter.

3.3 Data Acquisition

The rate at which data acquisition systems function may range from daily sampling to sampling in short MHz bursts and will depend very much on the signal being analysed, the information required from the signal and the technique used to extract data from the signal. Arnal (1977), Shaw et al (1983) and Castro (1984) used very fast analogue-to-digital conversion techniques to store turbulent and transitional hot wire signals in the form of digital data in the computer memory with subsequent analysis of this data to give values of mean velocity, rms of the fluctuating velocity component and, in the case of transitional flows, intermittency. To ensure that the computer reconstructs the correct waveform from the digital data it is required that the sampling rate is at least twice as fast as the highest frequency component in the signal (Shannon sampling theorem), hence the need for very fast ADC when digitising turbulence signals which may have frequency components as high as 2 kHz. (Jarvis (1985) suggested that for most engineering purposes the sampling rate should be at least 5 times the highest frequency component in the signal).
The advantage of storing a complete digital signal is that, provided the data is stored in a retrievable form, further analysis of the signal can be performed at a later date without having to reconstruct and remeasure the flow.

Apart from the expensive fast ADC required, a disadvantage of this technique, especially when using microcomputers, is the large amount of computer storage required to store a small sample of a digitised turbulence signal. For example; in the experiments carried out by Shaw et al (1983) the signal is being sampled at 20 kHz and since each data point or digital number occupies 2 bytes of computer memory then the 32 k bytes of memory (RAM) available on the BBC microcomputer would be completely allocated after 0.8s. In a transitional flow this would hardly be enough time to obtain a representative sample of the signal for subsequent processing.

Another method of obtaining information such as rms of the fluctuating velocity, intermittency, etc from the hot wire signal, which is more suited to the microcomputer, is to first pass the hot wire signal to analogue type instruments which measure the physical quantities required and output related voltages. The output voltages can then be passed, after conditioning, to the microcomputer via an analogue to digital converter as before. To increase the accuracy of this method a large number of samples from each device can be averaged with only the mean value being stored in the computer memory. The mean value of the fluid velocity can also be obtained by this method directly from the linearised hot wire signal by sampling at frequencies which can be much lower than those suggested by
the Shannon Sampling Theory, Arnal (1977). The reason for this is the quasi-steady nature of the hot wire signal from a transitional or turbulent flow, ie the frequency and amplitude of the signal are non-uniform but the signal does have a time steady average value. Therefore, provided the sample frequency is regular and a reasonable number of values are averaged, an accurate value of mean velocity will be obtained even for low sample rates.

When using a microcomputer this method has the advantage of addressing very little RAM for the storage of data as only mean values are actually committed to memory. The sampling rate does not need to be very fast as the signal is not being digitised, therefore cheaper ADCs can be used and can be accessed in a high level language such as BASIC giving the added advantage of simplifying the software.

Initial tests in a turbulent jet flow using the DISA 55M series equipment and a Cromenco Z - 2D microcomputer fitted with two different types of analogue-to-digital converters were conducted to confirm that reliable mean values of velocity and rms of the fluctuating velocity could be obtained from the hot wire signal in a highly turbulent flow using fairly slow sample rates.

The two analogue-to-digital converters used were a 12 bit dual slope integrating converter and an 8 bit successive approximation type converter. The 12 bit 3D INLAB R-12ADS dual slope converter is an integrating type and operates by charging a capacitor for a fixed time interval then a clock and binary counter are used to count the time taken for the capacitor to
discharge. The conversion rate of this type of converter is very slow. The converter being used has a conversion rate of only 5 Hz, but an advantage of this method, due to the integrating effect of the converter, is that the influence of high noise levels on a signal are eliminated.

The successive approximation converter operates on an entirely different principle. This is a counter type converter and its main components are a counter, a comparator and a Digital to Analogue Converter (DAC). When an analogue signal is fed into the converter the counter starts to count and passes a digital value to the DAC, starting with the most significant bit (MSB). The output from the DAC, is then compared to the analogue signal being measured and if the signal is greater than the output from the DAC the "1" in the MSB of the counter is retained. If the signal is lower then the "1" in the MSB of the counter is removed. This process is repeated until the DAC output compares with the analogue input signal. This type of converter has a much faster rate of conversion than the integrating type described above, typically 100 ms, but when accessing the ADC in Basic using the CROMEMCO Z - 2D micro, the maximum sample rate is only 30 Hz.

The initial investigation using the apparatus shown in fig. [3.3.1] with the 8 bit successive approximation converter demonstrated that the principle of averaging mean values from analogue devices by digitally sampling and averaging their outputs could be used successfully as excellent agreement was achieved between the instrument analogue displayed value and those obtained from the microcomputer, with the ADC system.
Surprisingly when using the 12 bit integrating ADC, the results obtained were poor. The values obtained by digitally sampling and averaging the signal were consistently below those read directly from a voltmeter. It was thought that this lack of agreement was due to the large conversion time required by this converter and it was concluded that this converter would be of little use when measuring rapid fluctuating signals, such as the signal from a hot wire probe in a turbulent flow.

3.4 Choice of microcomputer

As the microcomputer chosen was to be dedicated to this project, the main constraints on the choice were the cost, availability, and the fact that it was to be interfaced to the DISA 55M series hot wire equipment already available within the department. Although the Z - 2D cromemco microcomputer, used for the initial turbulent jet study described in the previous section, is a very powerful microcomputer which has the facility to be programmed in a number of high level languages such as FORTRAN and ALGOL as well as the usual micro language BASIC, it was not considered suitable for this project mainly because it was extensively used by undergraduates. This made it essentially unavailable, but it was also rejected because of its large physical size, the fact that it was not particularly reliable and had only modest graphics.

A wide range of smaller but more suitable microcomputers, which are relatively inexpensive, are now available on the market; one such computer is the BBC Microcomputer. Because of its growing popularity in educational establishments and the fact that it has an on board 4 channel, 12-bit analogue to digital converter and an easily accessible 8-bit user port to facilitate the control
of peripheral devices, it seemed a natural choice. The BBC microcomputer also has the advantage of an extended high level BASIC with excellent file handling facilities and colour graphics. A further asset of the machine is the ease by which commercially available hardware can be added-on to the computer, as areas of memory called FRED and JIM, addressed within the range &FCOO to &FDFF have been specifically reserved for such additions. Communication with these add-ons is via the 1M Hz expansion bus where the term 1 MHz simply refers to the speed at which it operates.

A disadvantage of the BBC microcomputer is the limited amount of RAM. This can be overcome however, by using a "dump-CHAIN-retrieve" routine. The BBC BASIC CHAIN command enables a program which is stored on disc to be called from a program being run in the computer memory. The procedure would be to dump relevant data from the initial program to disc, then CHAIN an extension program which is loaded into the machine memory over the original program. This extension program can then retrieve the data and continue with the analysis.

A complete microcomputer system based on the BBC-B microcomputer with a CUMANA 40/80 track switchable double disc drive, an EPSON FX-80 printer and a MICROVITEC colour monitor was purchased at a price of approximately £1100 (1983 prices) and incorporated into the wind tunnel test facility fig. [3.4.1]

Due to the unsuitability of the BBCs on board ADCs, see section (3.5), it was subsequently found necessary to extend this system by adding the BEEBEX Eurocard mini rack system fitted with the CUBAN-8 DAC card at a further cost of approximately £400. This system is described in detail in section [3.6].
After the BBC microcomputer had been purchased it was discovered that the built in four channel analogue to digital converter was an integrating type converter which had been shown previously (see Section 3.3) to be unsuitable for measurements in highly turbulent or rapid fluctuating flows. However the BBC *single slope* integrating converter, as described by Bannister & Whitehead (1985), operates at a rate twenty times faster than the (3D INLAB-R12ADS) *dual slope* integrating converter previously tested. For this reason it was decided to persevere further with the BBC on board converters with the knowledge that if problems were encountered more suitable add on ADC systems are available for use with the BBC microcomputer. The four ADC channels available on the BBC micro are accessed in high level BASIC by the command ADVAL (N) where N is the channel number, 1 to 4. This returns a 16 bit value with the four least significant bits set to zero and the true 12 bit number associated with the analogue signal can be obtained by ADVAL (N) DIV16. In actual fact, because of the low reference voltage (1.8V) and the high noise level on the ADC chip, only a 9-bit value can be obtained with any confidence (Beverley 1984). This is not a problem however, as initial tests showed 8-bit resolution to be satisfactory for measurements in turbulent flows although Beverley also showed that greater accuracy could be achieved at the expense of conversion time, if machine code averaging routines are used to reduce the standard deviation of the readings.
The speed at which conversion takes place on a single channel is 10 ms, although this cannot be realised if more than one channel is being used as conversion has to be complete at every channel before the values of any one channel can be read, effectively giving an overall conversion time of 40 ms if all channels are being used. The reason for this is that when the ADVAL command is made the four channels are scanned in reverse order and the values at each channel are not available until the end of conversion on the last channel has been sensed using ADVAL (0). For example; consider the program below to read in 10 values from each channel.

```
10 FOR K = 1 TO 10
20 REPEAT UNTIL ADVAL (0) DIV256 = 1
30 CH1%(K) = ADVAL (1) DIV16
40 CH2%(K) = ADVAL (2) DIV16
50 CH3%(K) = ADVAL (3) DIV16
60 CH4%(K) = ADVAL (4) DIV16
70 NEXT K
```

The REPEAT UNTIL statement ensures that conversion at channel 1, and hence all other channels since they are being scanned in reverse order, is complete before the values are available for reading. Channels can be switched off using the *FX16 command which will effectively speed up the scan rate and hence the rate at which values are available for reading. *FX16,1 will initialise channel 1 only, hence a sample rate of approximately 100 Hz ie 10 ms conversion can be achieved; *FX16,3 will initialise channel 3 but will also switch on channels 1 and 2 therefore a sample rate of approximately only 30 Hz can be achieved if three channels are being used. When accessing the ADC channels in BASIC therefore, channel 1 is the only channel which can sample at rates
close to the conversion rate of 10 ms. The rate at which additional channels can be sampled will be a 10 ms multiple of the number of channels switched on.

The ADC conversion rate can be increased further by switching the ADC chip from 12 bit mode to 8 bit mode giving a conversion rate of 4 ms per channel but this is very rarely used because the inherent error present in the 12 bit reading which reduces it to having only 9 bit accuracy is equally bad in the 8 bit reading reducing it to 5 or 6 bit accuracy (Beverley 1985). By far the most serious failing of the on board ADC system, and one which is very difficult to overcome, is the fact that the machine reference voltage, specified at 1.8v, is not constant. When the machine used for this project is powered up, the reference voltage has a value of 1.91v but over a period of about four hours this reduces by about 6% to a value of approximately 1.8v and still does not hold steady at this value but drifts between 1.8v and 1.83v. This is obviously not acceptable for precision data acquisition systems, although it can be allowed for in the software by continually feeding in the measured reference voltage. This is somewhat inconvenient and can introduce unnecessary errors.

After all these problems had been identified the author developed a distinct lack of confidence in the on board BBC-ADC system. This was justified when incorporating the on board ADC into the data acquisition system as it was found that a steady mean value of velocity could not be obtained from the hot wire signal of a steady freestream flow even when averaging 1000 values at the maximum sample rate. For this reason, and
the previously mentioned problems associated with the BBC-ADC port, a more effective data acquisition system was obtained employing a separate interface which connects directly into the microcomputer bus structure.

3.6 Accessing Signals using the BEEBEX Eurocard Extension

The interface chosen to enhance the data acquisition system was that termed BEEBEX, supplied by Control Universal of Cambridge, and is a general purpose Eurocard extension unit for the BBC micro. When incorporated into a mini rack system this becomes an extremely versatile method of expanding the BBC micro as a number of Eurocards which include analogue to digital converters, digital to analogue converters, digital i/o, heavy duty industrial opto-isolated i/o etc, become available as hardware extensions. The BEEBEX Eurocard mini rack is plugged into the 1 MHz expansion bus on the BBC micro and is controlled through a specific byte in memory reserved for the BEEBEX system. This is the last byte in the area of memory called FRED and is addressed at &FCFF.

As initial tests had shown that 8-bit accuracy was sufficient for the purpose of this investigation, it was decided to use 8-bit resolution analogue-to-digital conversion Eurocard termed CUBAN-8 for the use with the BEEBEX system. This card was developed jointly by Control Universal and Paisley College Microelectronics Educational Development Centre, see Ferguson et al (1981) for details. The CUBAN-8 card has 16 analogue input channels, 1 analogue output channel, 16 digital i/o channels contained in two 8-bit user ports, termed PORT A and PORT B, and four control lines, all available via a 40 way socket on the edge of the card. To simplify connection to these channels an interface was built which transfers the channel from the 40 way edge socket to 4 mm

60
jack plug sockets. This interface is shown in fig. [3.6.2]. The CUBAN-8 ADC is a successive approximation type, shown previously to be suitable for measurements in a turbulent flow, with an accuracy of ±1 bit and, when using BBC BASIC to access the ADC, has a sample rate of 500 Hz. (The conversion rate of the ADC is specified as 10,000 Hz but this cannot be realised when sampling in BASIC. To achieve sample rates close to the specified conversion rate machine code programs would have to be used).

The easiest method of accessing the CUBAN-8 card, with BBC software for reading data from a particular bit on an output port is to utilise a sideways ROM fitted into one of the spare sockets under the Keyboard of the BBC micro. Such a control ROM is supplied by Control Universal and is enabled in the software by the command *IO. When the *IO is initialised, other sideways utility ROMS, if fitted, such as the Disk Operating System are disenabled. Therefore, to use the SAVE, LOAD and CHAIN commands the Disk Operating System must be reinitialised using *DISK. The concept of *IO is that any area of memory outside the BBC micro is treated in the same way as a disc file using OPENUP, PTR# (position pointer), BGET# (Get Byte), BPUT# (Put Byte). When the PAGE and BLOCK switches (see fig. [3.6.2] for location of the PAGE & BLOCK switches on the CUBAN-8 Card) are set to Ø and C then the card is accessed by

\[
A\% = \text{OPENUP"CU-DAC8 \\&C000"}
\]

(the % symbol indicates integer values)

Consider the program overleaf to read in 10 values from channel 1, then 10 values from channel 5.
10 10 - initiate *10 ROM

20 CLOSE#0 - precautionary-close all opened files

30 A% = OPENUP"CU-DAC8 & C000 - access CUBAN-8 card

40 PTR#A% = 1 - set pointer to channel 1, pointer will stay at channel 1 until moved to another channel

50 FOR K = 1 to 10

60 Value %(K) = BGET#A% - Get value from channel 1

70 NEXT

80 PTR#A% = 5

90 FOR K = 1 to 10

100 Value%(K) = BGET#A% - Get value from channel 5

110 NEXT

The BEEBEX Eurcard system, accessed in BASIC with a sample rate of 500 Hz, was found to be completely satisfactory for the measurements of the flow variables and was a valuable addition to the data acquisition and control system developed.

3.7 Control of the hot wire probe position

The hot wire probe is positioned via a traverse mechanism and stepper motor connected to a DISA (52B01) Sweep Drive Unit (SDU) which is capable of being stopped during a sweep by closing an external switch. The CUBAN-8 card is fitted with a 6522 VIA which contains the 16 i/o digital channels in the form of two 8-bit user ports termed PORT A and PORT B and computer control of the SDU is achieved via the LSB of PORT B. When the LSB is set high a reed relay is energised and the switch closes to stop the SDU. The opposite occurs when the LSB is set low. Details of the reed relay interface are given in fig. [3.7.1].
Initially, the direction in which information is to travel over the bi-directional port has to be set up and this is done via the port Data Direction Register (DDR). Setting all the bits of the DDR to 1 (or High) causes all the bits of the user port to behave as outputs, and setting all bits of the DDR to 0 (or Low) causes all the bits of the user port to behave as inputs. A combination of inputs and outputs can be obtained by setting the relevant bits of the DDR to either 1 or 0.

For this application the LSB of Port B has to be set to output and this is done by placing a 1 in the LSB of the DDR. In actual fact all the bits of the user port were set to output by placing a 1 in every bit of the DDR, i.e. passing the value 255 to the DDR, but the status of the higher 7 bits of Port B is irrelevant as they are not used.

As before if the PAGE and BLOCK switches are set to 0 and C then PORT B of the CUBAN 8 card is addressed as "BUS &C000"

and the corresponding DDR is addressed as "BUS &C002"

A program to set all bits of PORT B as output, and output a logic '1' on the LSB is as follows:

10*10 - enable control ROM
20CLOSE#0 - precautionary: close all files
30ddr%=OPENUP"BUS&CO002" - access DDR
40BPUT#ddr%,225 - set all bits of user port to output
50CLOSE#ddr% - close DDR
60Pb%=OPENUP"BUS&C000" - access port B
70BPUT#Pb%,1 - output logic 1 or high level on LSB Port B
The fundamental measurements to be made in the present work are those of mean velocity, rms of the fluctuating velocity, intermittency and position normal to the plate surface. These measurements are obtained by sampling the analogue signal outputs from the relevant DISA hot wire instrumentation and passing them to the BBC micro via the CUBAN-8 ADC. To ensure the full range of the ADC is used, i.e. the full 255 bits for a maximum input of 2.5v, and also that the ADC is not overloaded, the maximum reading expected from each instrument must be conditioned to approximately 2.5v. This is done by passing the analogue outputs from the DISA instrumentation to a FYLDE modular instrumentation rack containing OpAmps having x0.1 and x1 switched gains with a x10 variable control and digital display monitor.

**Mean velocity:** The mean velocity is obtained by sampling and averaging the linearised hot wire signal which has been passed through a 2 kHz L.P. filter to eliminate electrical noise.

It is worth noting at this stage the reason why the hot wire signal is linearised directly using the DISA (55M25) analogue lineariser instead of linearising the probes within the computer software. It is known that the calibration of a hot wire probe adheres to Kings Law, equation 3.1:

\[ q^2 = q_0 + Bu \]

where \( q \) is the voltage from the hot wire anemometer

\( q_0 \) and \( B \) are constants

Therefore, it would have been a fairly simple task to linearise the probes within the software by a least square fit of Kings law to a set of calibration points in order to obtain
the constants \( p \) and \( B \). With the constants known this law can then be used to convert averaged values of voltage, obtained directly from sampling the non-linearised hot wire signal, to values of mean velocity.

However, in transitional boundary layer flows this presents problems which arise from the fact that the voltage readings from the hot wire signal are averaged before they are linearised. Dhawan and Narasimha (1957) pointed out that in a transitional flow the mean velocity obtained from averaging instrumentation is not the same as the true mean velocity. This is because the transitional mean velocity is a composite consisting of an intermittency weighted proportion of the laminar and turbulent velocity components, ie

\[
\bar{u}_L = (1 - \gamma) \bar{u}_L + \gamma \bar{u}_T \quad \ldots \ldots \ldots \ldots \quad 3.2
\]

In the case of a pitot tube, which is an averaging instrument, where the reading is proportional to the pressure, ie \( u^2 \) then

\[
\bar{u}_{pt} = \{(1 - \gamma) \bar{u}_L^2 + \gamma \bar{u}_T^2\}^{\frac{1}{2}} \quad \ldots \ldots \ldots \ldots \quad 3.3
\]

which is not the same as the true mean velocity given in 3.2.

The same difficulty is extended to measurements using a non-linearised hot wire probe where the reading from the hot wire anemometer is basically proportional to \( u^4 \). This difficulty is overcome if the signal from the hot wire is linearised directly by passing it through the DISA 55M25 lineariser thereby obtaining a voltage reading which is directly proportional to the fluid velocity, enabling the signal to be sampled and averaged to give a true mean velocity.

The signal from the hot wire was linearised such that the voltage output from the DISA 55M25 lineariser was equivalent to \( 1/10 \)th of the fluid velocity. The maximum velocity expected in
the planned experiments was approximately 20 m/s which would correspond to an output of 2v from the lineariser, therefore to use the full range of the ADC the output from the lineariser was passed through an amplifier, on the FYLDE instrumentation rack, set at a value of x1.25. A digital value of 255 as read by the computer will now correspond to a fluid velocity of 20 m/s and to convert this digital value back to a velocity for use with the computer software for subsequent processing or display a calibration constant is required which is calculated from

\[
\text{max. fluid velocity} = \frac{\text{calibration constant}}{255} \times 255 \quad \ldots \quad 3.4
\]

in this case

\[
\text{calibration constant} = \frac{20}{255} = 0.07843 \text{ (m/s)/bit}
\]

**rms of velocity fluctuation** - The rms of the velocity fluctuation is obtained by passing the linearised hot wire signal through the DISA 55D35 RMS voltmeter which has a twelve position rotary switch to select a number of measurement ranges varying from 0 to 1 mv to 0 to 300v fsd and has an analogue output of 1v for fsd which is linearly related to these ranges. The analogue output value can increase to a value of 1.2v if the scale is overloaded, ie the incorrect range is selected, and for this reason the specified output voltage of 1v for fsd is not conditioned to 2.5v, to utilise the full range of the ADC, but only conditioned to 2v to prevent an overload condition damaging the ADC.

The signal is conditioned, as before, by passing it through an amplifier on the FYLDE instrument rack set to a value of x2 giving a digital value of 204 for fsd of the rms meter. The calibration constant is calculated depending on the range selected.
rms range = calibration constant x 204 ............ 3.5

(the factor of 10 multiple of the rms range is to convert the rms voltage to an rms velocity since the voltage is linearised to correspond to 1/10th of velocity).

**Intermittency** - Intermittency was measured using the apparatus shown in fig. [2.8.1] and described in section 2.8. Unfortunately the averaging DVM used for visual display of the intermittency function does not have an analogue output therefore the signal from the DISA 52B10 comparitor, which outputs discrete 5v square pulses, was passed to a true integrator DISA 52B30 set on a low integration time (0.5s). The signal from the true integrator outputs a maximum value of 5v for \( \gamma = 1 \) therefore the signal from this was conditioned, by passing it through a x0.5v amplifier, to give a maximum output of 2.5v utilising the full range of the ADC and preventing overload. A digital value of 255 as read by the computer will correspond to an intermittency of 100% or \( \gamma = 1 \) and the calibration constant can be calculated as shown previously.

**Vertical Positioning of the hot wire probe** - The position of the probe above the plate is determined from the output voltage of the DISA 55D35 Sweep Drive Unit which is basically a variable D.C. ramp generator, the output of which is made proportional to the linear displacement of hot wire probe via a stepper motor and traverse mechanism, fig. [2.1.2]. Calibration of the sweep drive unit, fig. [3.8.1] gives a linear relationship between the voltage and the displacement.

\[
y = y_0 + K(V - V_0) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.6
\]

where \( y \) is the vertical displacement, in mm, corresponding to \( V \),
the displacement voltage and the suffix "o" denotes the datum values. From the calibration on fig. [3.8.1] the value of $K = 10.52 \text{ mm/volt}$ and as the maximum traverse of the probe in the wind tunnel working section is limited by geometry to approximately 50 mm, the position signal was passed through a $x0.5$ amplifier giving a maximum displacement of 52.5 mm corresponding to a digital value of 255 as read by the computer. The calibration constants for reconversion of the digital value to position can be calculated from this as before.

A schematic layout of the complete Data acquisition and control apparatus is shown in fig. [3.8.2].

3.9 Development of Data Acquisition and Control Software

This section details the development of the software for the basic data-acquisition and control system used for the measurement of the mean velocity profiles as well as two data acquisition programs for the measurement of the intermittency and freestream turbulence distributions along the plate.

**Data acquisition and control code for measurement of mean velocity profiles** - The object was, for a particular location on the plate surface and mainstream velocity, to measure the flow variables - mean velocity, rms of the fluctuating velocity and intermittency - at specified step increments through the boundary layer, measured relative to a datum, until the freestream velocity was reached. The probe datum position was set manually using a scaled block placed behind the probe and viewing the probe and block through a cathetometer from outside the tunnel working section. The software was developed to automatically control the experiment from this datum point until a complete boundary layer traverse
had taken place.

Firstly the output voltage from the DISA Sweep Drive Unit was read, via the ADC, to determine the digital value corresponding to the probe datum and all other probe positions were calculated relative to this. A number of readings of mean velocity, rms of velocity fluctuation and intermittency were then accessed by the computer, via the ADC, from the relevant instruments before their averages were stored in specified arrays and the probe was moved to the next position. The probe movement was controlled, as described in more detail in section 3.7, by outputting a control signal via the LSB of user port B on the CUBAN-8 6522 VIA. Setting this bit low switches on the sweep drive unit while setting it high switches off the sweep drive unit. Therefore, once all the values have been stored, the LSB of port B is set low thus moving the probe. While the probe is moving the output voltage from the sweep drive unit is monitored by the microcomputer until the output exceeds the value of the datum plus the specified step increment and at this point the LSB of port B is set high and the probe traverse is stopped. The BASIC code for this is:

10 BPUT#Pb,0  - LSB of Port B Set Low
20 REPEAT UNTIL BGET#0> (Datum+Step Inc) - read in value from channel 0 until > (LIMIT)
30 BPUT#Pb,1  - LSB of Port B Set High

The actual position in which the probe stopped is then determined by reading in and averaging a number of values of output voltage from the sweep drive unit. The flow variables are then read in again and, once averaged values of each variable have been stored, the probe is moved to the next position. This continues until the freestream velocity is reached, sensed by three consecutive values of mean
velocity being within ±0.5% of each other, and the probe traverse is stopped. The freestream velocity is then determined by averaging the mean velocities at the last three positions and the boundary layer thickness is estimated, as the $y$ value corresponding to 99.5% of the freestream velocity, by a linear interpolation routine. Data such as ambient pressure and temperature, distance of the probe from the plate leading edge and spanwise probe position are fed in interactively at the start of the program. This data along with values of the freestream velocity, the boundary layer thickness and values of $y/\delta$, $u/U_\infty$, $\gamma$, and rms velocity obtained for each step increment are printed out and then dumped to a disc file. A typical printout of this data is shown in fig. [3.9.1]. A graphics program is then 'CHAINed' which retrieves the data from the disc file and displays the mean velocity data on axes of $y/\delta$ against $u/U_\infty$ along with the Blasius and $1/7$th Power Law profiles for comparison purposes, as illustrated in fig. [3.9.2].

Because of the contrasting shape of the velocity profiles in laminar and turbulent boundary layer flows the program provides the facility to choose the step increments for the upper and lower regions of the boundary layer. These step increments are fed in interactively at the start of the program.

To increase the accuracy at the flow variables 100 values of position, 5000 values of mean velocity (corresponding to a 10s sample time), 1000 values of rms velocity and 1000 values of the intermittency factor were averaged before storing the mean values for one particular point. This takes in excess of 15 seconds.
and for a typical boundary layer, with say 20 step increments, a complete traverse would take approximately 5 minutes.

A flow diagram of this program is shown in fig.[3.9.3] and a printout is included in Appendix 5.

**Data acquisition Codes for streamwise freestream turbulence and intermittency distributions** - Two data acquisition and operator interactive programs, which do not involve any element of control, were developed to give large sample times for obtaining accurate values of freestream turbulence and intermittency. The intermittency program prompts the operator to position the probe close to the place surface and input the streamwise position of the probe then press RETURN for the values to be read in from the intermittency instrumentation. (10,000 values are read in and averaged giving a mean value of intermittency over a period of approximately 20 seconds). Once the values have been read in and the average value stored in an appropriate array the program prompts the operator to move the probe to the next measurement station and press RETURN again, to read in the values. This continues until the operator is satisfied the run is complete and then presses the 'C' key for this data to be dumped to disc. The data is stored on disc in the form of \(x\) and \( \gamma \) values and can be retrieved at any time for subsequent processing.

The freestream turbulence distribution program operates in a similar manner but for this case the probe is placed in the freestream and the rms of the velocity fluctuation and the mean velocity are read in and the freestream turbulence is calculated from

\[
T_u = \frac{\sqrt{u'^2}}{U_\infty} \times 100
\]

A printout of both these programs is given in Appendix 5.
The auxiliary unit is used to filter the hot wire signal (2 kHz L.P.) and to remove the zero flow voltage.

Fig. 3.3.1 Schematic layout of turbulent jet flow test
Fig. 3.4.1 Photograph of tunnel working section with BBC System below

Fig. 3.6.2 CUBAN-8/4mm socket interface
Fig. 3.6.1 Schematic layout of CUBAN-8 card showing position of PAGE and BLOCK
Fig. 3.7.1 stepper motor on/off interface
Fig. 3.8.1. Calibration of DISA 52B01 sweep Drive Unit

Slope = -0.095 volts/mm
Fig. 3.8.2 Schematic Layout of Data Acquisition and Control System

- Stepper Motor
- Probe
- Plate
- CTA
- Lineariser
- 2 kHz LP Filter
- DVM
- Sweep drive Unit
- RMS
- 50,100,200 Hz HP Filter
- Signal Modifier
- Amplitude Comparator
- True Integrator
- DVM
- RGB Monitor
- Disc Drives
- BBC Microcomputer
- BEEBEX
- Printer

Schematic Layout of Data Acquisition and Control System
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<tr>
<th>Velocity (m/s)</th>
<th>Y-Pos. (mm)</th>
<th>Intermittency</th>
<th>RMS-Vel. (m/s)</th>
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DIST. FROM L.E. = 1000mm  
SPANWISE LOCATION = 0mm  
APPROX. EDGE OF BOUNDARY LAYER = 13.18mm  
FREE STREAM VELOCITY = 18.59mm

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<th>y/d</th>
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EYEBALL AVE OF INTERMITTENCY AT y/d=0.2 = 0

AVE. OF INTERMITTENCY VALUES BELOW (y/d=0.2) = 0.98

Fig. 3.9.1 Printout from Data Acquisition & Control Prog.
Fig. 3.9.2 Dump from $y/\delta_y$ vs $u/U_\infty$ graphics program.
Fig. 3.9.3 Flow Diagram of Data Acquisition and Control Program
4.1 Introduction

In this chapter, the methods used for reducing the mean velocity profile data for the laminar, turbulent and transitional boundary layers are presented along with some estimation of the errors involved. The technique used for determining the start and end of the transition region is discussed and compared to methods used by other researchers. Also, the early development of the turbulent boundary layer, associated with transitional boundary layers, is considered with reference to low Reynolds number effects. Finally, the two dimensionality of the boundary layer flows are examined and the momentum balance technique for testing two dimensionality is described.

4.2 Reduction of Laminar Mean Velocity Profiles

The Pohlhausen (1921) solution for the laminar boundary layer, in arbitrary pressure gradients, is based on the assumption that the laminar boundary layer velocity profile can be represented by a fourth order polynomial of the form.

\[ \frac{\bar{u}}{U_\infty} = A \left( \frac{y}{\delta} \right) + B \left( \frac{y}{\delta} \right)^2 + C \left( \frac{y}{\delta} \right)^3 + D \left( \frac{y}{\delta} \right)^4 \]  

A least squares technique was used to fit a polynomial through the data points of \( \frac{u}{U_\infty} \) against \( \frac{y}{\delta} \) and with the constants known the boundary layer integral parameters \( \delta^* \), \( \theta \) and \( \delta^{**} \) were easily obtained by direct integration of the respective functions. The shape factors \( H_{12} \) and \( H_{32} \) immediately follow from the integral parameters. (In actual fact it was found that a third order
polynomial was sufficient to fit the data. Therefore, this was used in preference to a fourth order polynomial as it simplified the software). To determine the wall shear stress from the profile data available, use was made of the fact that in the laminar layer the shear stress is directly proportional to the rate of strain.

\[ \tau_0 = \mu \frac{du}{dy} \]  

and in the region \( 0 < \frac{\bar{u}}{U_\infty} < 0.45 \) the slope \( \frac{du}{dy} \) is approximately linear. The shear stresses was determined therefore, by averaging the slope of all the data points in this region. The thickness of the boundary layer was defined as the \( y \) value corresponding to \( \bar{u} = 0.995 U_\infty \) and was determined via a linear interpolation routine. A typical printout from this analysis is shown in fig. [4.2.1] and a fit of the third order polynomial to a set of data is shown in fig. [4.2.2]. A printout of the program used for this analysis is given in appendix 5.

4.3 Reduction of turbulent mean velocity profiles

The analysis of Coles (1968), as applied to the data presented at the Standford Conference, Coles & Hirst (1968), was used for the reduction of the turbulent mean velocity profiles measured. A brief description of this analysis is given below.

It is generally accepted that the turbulent boundary layer consists of an inner and outer region. In the inner region, which contains but extends far beyond the laminar sublayer, viscous and turbulent stresses are important, while in the outer region the turbulent stresses dominate. A schematic representation of the turbulent boundary layer is shown in fig. [4.3.1].
In the viscous sublayer, which accounts for approximately 1% of the total shear layer thickness, viscous forces dominate and the mean velocity profile can be approximated by:

$$\frac{\bar{u}}{u_\tau} = \frac{y u_\tau}{v}$$  \hspace{1cm} 4.3

or

$$u^+ = y^+$$

Outside this viscous sublayer the analysis of Coles assumes that the turbulent boundary layer can be modelled by two separate wall and wake functions. This is shown schematically below in fig. [4.3.2].
The wall function can be obtained from Prandtl's mixing length concept assuming that the mixing length $l$ is proportional to $y$ close to the wall (i.e. $l = ky$) and that the wall shear stress $\tau_0 = \rho l \left( \frac{\partial u}{\partial y} \right)^2$ remains constant.

$$\cdot \cdot \cdot \tau_0 = \rho \left( k^2 y^2 \left( \frac{\partial u}{\partial y} \right)^2 \right) \quad \cdots \cdots \ 4.4$$

$$\frac{\partial u}{\partial y} = \frac{u_\tau}{ky} \quad \cdots \cdots \ 4.5$$

which integrates to give

$$\bar{u} = \frac{u_\tau}{k} \ln(y) + \text{constant} \quad \cdots \cdots \ 4.6$$

or in non-dimensional form

$$\frac{\bar{u}}{u_\tau} = \frac{1}{k} \ln \frac{yu_\tau}{v} + C \quad \cdots \cdots \ 4.7$$

Equation 4.5 can also be derived from dimensional analysis arguments. Cebeci & Bradshaw (1977) & Bradshaw (1972).

The constants $k$ and $C$ are wholly empirical and their values will be discussed later in section 4.5.

The wake function was obtained by relating the outer mean velocity profile to the inner profile and defining the function such that it represented the deviation from the law of the wall (i.e. equation 4.7).

The form of the wake function obtained by Coles (1956) was

$$W = 2 \sin^2 \left( \pi / 2 \cdot y / \delta \right) \quad \cdots \cdots \ 4.8$$
Therefore the composite turbulent boundary layer outside
the viscous sub layer, (normally outside $\frac{y u}{u_T} > 50$) is:

$$\bar{u} / u_T = \frac{1}{k} \ln \left( \frac{y u_T}{v} \right) + C + \frac{2\Pi}{k} \sin^2 \left( \frac{\nu}{2} \cdot \frac{y}{\delta} \right) \quad \ldots \quad 4.9$$

where $\Pi$ is a wake parameter related to the strength of the
wake function. The strength of the wake function $\Delta \bar{u} / u_T$
was defined by Coles as the maximum deviation of the velocity
profile from that obtained by equation 4.7 at the edge of the
boundary layer (ie at $y = \delta$) and can be calculated from:

$$\Delta \left( \frac{\bar{u}}{u_T} \right) = \left( \frac{\bar{u}}{u_T} \right) - \frac{1}{k} \ln \left( [y+] \right) + C = \frac{2\Pi}{k} \quad \ldots \quad 4.10$$

The analysis of Coles assumes that the law of the wall is
universally valid in the region $100 \leq y + \leq 300$ irrespective
of the freestream turbulence or any external pressure gradient
acting on the boundary layer. (In strong adverse pressure
gradient ie flows close to separation the law of the wall breaks
down and the above will not apply). Therefore, equation 4.7 was
iterated to obtain an optimum value of $u_T$ for each data point
within the specified region and the averaged value obtained was
taken as representative of the particular velocity profile being
analysed. The fitting region used by Coles was altered to
$y + > 30$ and $\frac{Y}{\delta} < 0.2$, on the suggestion of Murlis et al (1982),
to account for the low Reynolds number flows associated with
the present work. The boundary layer thickness, $\delta$, was then
determined from the data by the linear interpolation routine
used previously for the laminar profile analysis. With these
two values known, ie $u_T$ and $\delta$ the value of $\Pi$ can be obtained
from equation 4.9. Coles used a set of standard integrals for
the integral parameters within the viscous sub-layer ie from \( y = 0 \) to \( y^+ = 50 \). These were

\[
\begin{align*}
\int_0^{50} \left[ \frac{\bar{u}}{u_T} \right] \, d \left[ \frac{y u_T}{v} \right] &= 540 \\
\int_0^{50} \left[ \frac{\bar{u}}{u_T} \right]^2 \, d \left[ \frac{y u_T}{v} \right] &= 6546 \\
\int_0^{50} \left[ \frac{\bar{u}}{u_T} \right]^3 \, d \left[ \frac{y u_T}{v} \right] &= 82770
\end{align*}
\]

Equation 4.9 is then assumed to continue the integration from \( y^+ = 50 \) to some point in the log-law region. This point was taken to be the third data point ie \( y^+(3) \) in the analysis. In some cases the value of \( y^+(3) \) was actually less than 50 and for these cases the first data points are automatically deleted and the data renumbered. Integration from the third data point to the freestream was carried out by a parabolic fitting routine using a modified Simpson's rule. A parabola was fitted through three adjacent points, and the integrals from the first to second and from the second to third points are computed. The central point is then moved one point outward and the process repeated. The two values for each interval are then averaged providing an element of smoothing for the integrals. The integrals of

\[
\left( \frac{\bar{u}}{U_m} \right), \left( 1 - \frac{\bar{u}}{U_m} \right), \left( \frac{\bar{u}}{U_m} \right)^2 \text{ and } \left( \frac{\bar{u}}{U_m} \right)^3
\]

in this manner are appropriately combined to obtain the required integral thicknesses.
The wall shear stress, $\tau_0$, was calculated from the value of
$u_T$ obtained from the profile analysis, ie from velocity profile
measurements in the logarithmic region.

$$\tau_0 = \rho u_T^2$$

and the skin friction coefficient follows from

$$Cf = 2 \frac{\tau_0}{\rho U_\infty^2} = 2 \left( \frac{u_T}{U_\infty} \right)^2$$

The justification for using the log-law region for calculation
of $u_T$ is that the log-law is extraordinarily insensitive to the
variation of freestream turbulence and pressure gradient effects.
Evidence for its applicability is provided by the quality of the
fit of the present profiles, and of the vast amount of data
presented at the standard conference, to the universal logarithmic
law.

The value of $Cf$ can also be obtained by substitution of
the shape faction, $H_{12}$ and momentum thickness, $\theta$ from the profile
analysis into any one of a number of correlations of the form

$$Cf = f(R_0, H_{12})$$

Two such correlations are those of Ludwieg & Tillman

$$Cf = 0.246 R_0^{-0.268} \exp(-1.561H_{12})$$

and a curve fit due to White for the skin friction relation
derived from the wake integrations of Coles formula - equation 4.9.

$$Cf = \frac{0.3 \exp(-1.33H_{12})}{(\log_{10} R_0)^{1.74+0.31H_{12}}}$$
Preston tube measurements were also made, for a limited number of profiles, mainly for comparison with the values obtained by the methods mentioned above.

Taking the log-law values obtained from the velocity profiles as a reference, the deviation of the skin friction as measured by the other methods are:

<table>
<thead>
<tr>
<th>Method</th>
<th>Tu=0.5%</th>
<th>Tu=1.5%</th>
<th>Tu=1.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ludwig/Tillman</td>
<td>-2.2%</td>
<td>-0.6%</td>
<td>+1.0%</td>
</tr>
<tr>
<td>White-Coles</td>
<td>-5.8%</td>
<td>-4.0%</td>
<td>-3.25%</td>
</tr>
<tr>
<td>Preston tubes</td>
<td>-3.4%</td>
<td>-1.15%</td>
<td>-5.24%</td>
</tr>
</tbody>
</table>

A computer printout from the velocity profile analysis is given in fig. [4.3.3] and the universal turbulent boundary layer velocity profile is shown for a typical set of data along with the composite profile, defined by equation 4.9, in fig. [4.3.4]. A printout of the program used for the analysis is included in appendix 5.

4.4 Estimation of errors in boundary layer integral thicknesses

When linearising the signal from a hotwire probe using the DISA 55M25 lineariser there is an inherent parabolic error, usually with a maximum close to the centre of the linearised region and tailing off to zero at the maximum and minimum velocities, as shown in fig. [4.4.1].
Fig. 4.4.1 Lineariser error

This is only a very small error, usually of the order +1%, and as can be seen from an actual probe calibration, fig. [2.6.3], it is barely detectable. However, the error analysis in Appendix 1 shows that for a maximum error, \( em \), of +1% the corresponding error in displacement and momentum thickness are -2.52% and -2.24% respectively for a \( 1/7 \)th power law turbulent velocity profile and -0.93% and -0.78% respectively for a parabolic laminar type velocity profile. Possible error introduced into the boundary layer thicknesses which can be associated with the curve fitting and integration techniques, described in the previous two sections, are very difficult to define. For the laminar analysis, since the polynomial is integrated directly, all the likely error can be attributed to the fit of the polynomial to the data. As in most cases the polynomial fits the data very well, the errors are assumed to be minimal. For the turbulent boundary layer the analysis is the same as that used at the Stanford Conference. This is a well tested method and is considered to be as good, if
not better than any other methods available. Fraser (1979) compared integral thicknesses obtained by the Stanford Conference method with those obtained by planimeter measurements and found agreement to be within 0.7%.

4.5 Approach to equilibrium and low Reynolds number effects

The present investigation, although mainly concerned with the development of the transitional boundary layer, is also related to the early development of the turbulent boundary layer and the low Reynolds number effects associated with this early development. In order to explain the low Reynolds number effect, reference is made to the approach of a constant pressure turbulent boundary layer towards equilibrium conditions. Initially though, it is necessary to define an equilibrium turbulent boundary layer.

Bradshaw (1972) described a turbulent boundary layer as being in equilibrium if the generation of Reynolds stresses by interaction with the mean flow and existing Reynolds stresses is equal to the destruction of the Reynolds stresses by viscous forces. In contrast, self preserving boundary layers (often misleadingly called equilibrium layers) are defined by Townsend (1965) as those in which distributions of the flow quantities, ie Reynolds stresses, mean velocity etc have the same form at all distances from the flow origin differing only in common scales of velocity and length. The importance of self preserving flows is that rates of change of velocity and length scales can be predicted with no more specific assumption about the nature of turbulent motion than that the large scale motion is independent of fluid viscosity.
Clauser (1954) found that constant pressure turbulent boundary layers possessed a set of similar profiles when expressed in terms of the velocity defect. ie

\[ \frac{U_\infty - u}{u_T} = f \left( \frac{y}{\delta}, \Pi \right) \]

\( u_T \)

(This can be obtained from equation 4.9 by setting \( \bar{u} = U_\infty \) at \( y = \delta \) and subtracting the resultant equation from 4.9). He also reasoned that since such a set of profiles existed then the turbulent boundary layer in a constant pressure flow was indeed in equilibrium, (equilibrium layers in the Clauser sense are actually self preserving layers). Clauser went on to show, with some considerable experimental effort, that a turbulent boundary layer with variable pressure gradient but constant history, expressed in terms of the pressure gradient parameter, \( \beta = \frac{\delta^*}{\delta} \cdot \frac{du}{dx} \), also possessed such a set of similarity profiles. From this he concluded that such layers were also in turbulent equilibrium since the gross properties of the boundary layer could be expressed in terms of a single parameter: eg

\[ \frac{U_\infty - \bar{u}}{u_T} = f \left( \frac{y}{\delta}, \beta \right) \]

\( \Pi \) and \( \beta \) must be related and that \( \Pi \) and \( \beta \) must be constant in an equilibrium boundary layer. Coles (1962) found \( \Pi = 0.55 \) for a constant pressure boundary layer at Reynolds numbers, based on the momentum thickness, above 5000, but at Reynolds numbers below this the velocity defect factor \( \Delta \frac{\bar{u}}{u_T} = \frac{2}{k} \Pi \) was Reynolds number dependent.
Coles expressed this dependency through the wake function $II$ which has been conveniently curve fitted by Cebeci & Smith (1974) as:

$$II = 0.55 \left[1 - \exp(-0.243Z^{0.5} - 0.298Z)\right] \quad \text{...... 4.17}$$

where $Z = \left(\frac{R_\theta}{425} - 1\right)$

Simpson (1970) on the other hand correlated the velocity data in the outer similarity law by varying the Von Karman constant, $k$, and the additive constant in the law of the wall. From this he suggested that the Von Karman constant was Reynolds number dependant and should be replaced by the term $\Omega$

$$\Omega = 0.4 \left(\frac{R_\theta}{6000}\right)^{-\frac{1}{78}} \quad \text{...... 4.18}$$

for $R_\theta < 6000$.

In order to settle the controversy, Huffman and Bradshaw (1972) after critically reviewing the available literature and examining the low Reynolds number effect, reached the conclusion that $k$ was in fact constant, (equal to 0.41) and that the additive constant could be considered mildly Reynolds No. dependant. The Reynolds number effect was attributed to the affect of the turbulent-irrotational interface on the outer law of the wall and was substantiated by the fact that no such effects were present in duct flows which do not have such an interface. This in effect vindicates the observations of Coles (1962).

Since Huffman & Bradshaw concede that the additive constant, $C$, can vary and does in fact increase for $R_\theta$ values below approximately 1000, the value of 5.2, as suggested by Murlis (1975) and used by other researchers since then eg Castro (1984),
Fraser (1980), was adopted for C. This value was used in preference to the usual value of 5.0, as the present investigation is associated with the early development of the turbulent boundary layer and consequently \( R_0 \) values less than 1000 are fully expected.

Although the above low Reynolds number effect has been described for the special case of a constant pressure turbulent boundary layer, which is approaching equilibrium, all turbulent boundary layer flows with \( R_0 \) values less than approximately 5000 will be subject to this effect.

4.6 Transitional mean velocity profiles

The transitional boundary layer is characterised by regions of laminar and turbulent flow with the mean velocity at any height in the boundary layer defined by a near \( \bar{u} \) intermittency weighted average of the laminar and turbulent velocity contributions ie

\[
\bar{u}_t = (1 - \gamma) \bar{u}_L + \bar{u}_T
\]

Dhawan & Narasimha (1957) noted that although \( \gamma \) varies across the boundary layer, for the purposes of the profile calculation, the value of \( \gamma \) measured close to the wall, \( \gamma / \delta < 0.2 \), gives sufficiently accurate results for the whole profile. The intermittency distribution through the boundary layer, characterised by the variation \( \gamma(x) \), has only a secondary influence on the transition flow, \( \gamma(x) \) being the significant property. Since the transitional boundary layer is a composite consisting of laminar and turbulent velocity components, neither of the two analyses described in sections 4.2 and 4.3 are strictly applicable. However, since they are both purely numerical techniques, with a polynomial being fitted to the data in the
laminar analysis and a curve fitting and integration technique used for the bulk of the data in the turbulent analysis, true transition integral thicknesses can be representatively obtained. To enable the decision as to which analysis should be used a computer program was developed to display the measured data graphically, in ordinates \( \frac{y}{\delta} \) vs \( \frac{\bar{u}}{U_{\infty}} \), along with the third order polynomial fit to the data, on the R.G.B. monitor. If the polynomial was a good fit to the data, usually the case for values of \( \bar{y} < 0.5 \) where the boundary layer is dominantly laminar, the laminar analysis is used, otherwise the turbulent analysis is selected.

The skin friction coefficients calculated from these analyses, in the transitional boundary layer, will not give representative transitional values. The laminar analysis assumes equation 4.2 to be valid and does not account for the substantially larger contribution from the turbulent regime to the overall skin friction in the transitional boundary layer, hence the skin friction will be underestimated. Similarly in the turbulent boundary layer the skin friction coefficient will be overestimated.

In attempt to give a better account of the transitional local skin friction coefficients Fraser (1980) developed an empirical relation in the form

\[
C_{f_t} = f(R_\theta, H_{12}, \bar{y}) \quad \ldots \ldots \quad 4.19
\]

This relationship was devised from the observations of Emmons (1951) that the skin friction coefficient could be represented by

\[
C_{f_t} = (1 - \bar{y}) C_{f_L} + \bar{y} C_{f_T} \quad \ldots \ldots \quad 4.20
\]
The laminar skin friction component was obtained from the Thwaites (1949) solution and the turbulent component was derived by equating the turbulent velocity profile described by a power law, with the exponent n free, to the log-law relation given by 4.7 which was assumed to be universally valid at $y^+ = 100$.

This gives after some algebraic manipulation:

$$Cf_t = \frac{2 f_1(\lambda g)}{R_{FL}} (\frac{2}{(H_T-1)}) + 2\gamma \left[ \frac{R_{GT} K}{100} \times \frac{H_T (H_T+1)}{(H_T-1)} \right]^{\frac{-2(H_T-1)}{(H_T+1)}}$$

The laminar and turbulent component values cannot obviously be determined from experimental measurements in the transitional boundary layer. Therefore the values of $R_{GT}$ and $H_T$ are used in the formula and were found to give reasonable results. (See Fraser (1979) for a complete derivation of 4.21).

4.7 Determination of start and end of transition

For the purpose of the present study a reliable method of determining the position of the start and end of transition is of paramount importance. The method adopted was to place the hot wire probe close to the plate surface and pass the signal from the hot wire anemometer to the intermittency measurement apparatus. The start of transition was defined as the x position corresponding to a reading of $\gamma = 0.01$ and the end of transition as the x position corresponding to a reading of $\gamma = 0.99$. The signal from the hot wire anemometer was also passed to an oscilloscope and a loud speaker to provide audible and visual detection of the appearance of turbulent bursts or spots.
The method of detection of transition is very important as different techniques can give widely varying results for the position of start and end of transition. Hall & Gibbings (1972) noted that it would not be unreasonable to expect scatter of ±5% in the value of $R_{QS}$ due to the different detection methods. The present method is likely to give $R_{QS}$ values lower than those obtained by the common surface pitot method used by Hall & Gibbings and many others. However, the author suggests that the present method is more reliable and repeatable as it measures the intermittency function directly and does not lend itself to the degree of estimation required by other techniques. Sharma et al (1982) used a similar technique of measuring the intermittency function directly using flush mounted hot film probes, but they defined the start of transition at $\gamma = 0.1$ which would give values of $R_{QS}$ higher than those presented here. Provided the method used by other researchers is noted, then a comparison of results can be made by estimating the affect that the detection method has on determining the transition position.

4.8 Flow two dimensionality

Most of the boundary layer prediction methods assume that the flow is two dimensional in order to simplify the fundamental Navier-Stokes equations governing fluid motion, and enable a solution to be achieved. However, many of the early researchers, when obtaining data to validate such prediction methods, paid very little attention to the two-dimensionality of the flow and merely assumed this to be the case. In practice however, it is very difficult to obtain two dimensionality especially
in adverse pressure gradients. In 1954 Clauser reported, after much experimental effort in obtaining a two dimensional boundary flow in an adverse pressure gradient, quote - "we came to have great respect for the ease with which air can move laterally in boundary layers subject to adverse pressure gradients". Since the 1968 Standford Conference the importance of obtaining good quality two dimensional test flows has been realised and various methods have been developed to assess the quality of the flow with regard to two dimensionality, Fraser (1986).

One such method, which has been used in the present work, is the momentum balance principle. This method was used to assess the two dimensionality of the flows at the Standford Conference and consists basically of integrating with respect to x, the von Karman momentum equation in the form given below

\[ \frac{d}{dx} \left( \frac{U_\infty^2 \theta}{2} + \frac{\delta^*}{2} \right) - \frac{\tau_0}{\rho} = \frac{d}{dx} \left( \frac{U_\infty^2}{2} \right) \quad \ldots \ldots \quad 4.24 \]

normalising and integrating from \( x = x_i \) to \( x = x \), where subscript \( i \) denotes initial value, results in

\[ \frac{U_\infty^2 \theta}{(U_\infty^2 \theta)_i} - 1 + \int_{x_i}^{x} \frac{\delta^*}{\theta_i} d \left( \frac{U_\infty}{U_{\theta_i}} \right)^2 = \int_{x_i}^{x} \frac{U_T}{U_{\theta_i}} \left( \frac{x}{\theta_i} \right)^2 \quad \ldots \ldots \quad 4.25 \]

A computer program was used to determine the values of the left and right hand side of equation 4.25 using input data of \( x, \theta, u_T, U_\infty \) and \( \text{Cf} \) at various spanwise positions along the plate centre-line. The modified Simpsons rule described in section 4.3 was used to evaluate the integral terms and give a degree of smoothing to the data. Any lack of agreement between left and right hand sides of equation 4.25, termed PL & PR respectively,
indicating a lack of two dimensionality of the flow, assuming
the input values of $\theta$, $C_f$, $u_T$ and $U_\infty$ are confidently known.

Flow divergence is represented by $PR$ being greater than
$PL$ and flow convergence is represented by $PL$ being greater
than $PR$. The momentum balances of three test flows in favourable,
zero and adverse pressure gradients are shown in fig. [4.8.1].
Excellent agreement indicative of good two dimensionality is
obtained in the zero and favourable pressure gradient cases
with only a very slight convergence detectable. The adverse
pressure gradient, as expected, is not as good as the other two
cases but is still better than normally accepted two dimensional flows
such as Weighardt's flat plate flow, presented at the Standford
Conference as one of the better two dimensional flows.

The slight convergence of the flows is thought to be caused
by the side wall contamination as described in section 2.3.
**LAMINAR BOUNDARY LAYER INTEGRAL PARAMETERS**

- Approx edge of boundary layer: 4.95
- Displacement thickness: 1.61
- Momentum thickness: 0.67
- Energy thickness: 1.05
- Shape factor H12: 2.42
- Shape factor H32: 1.58
- Momentum th. Reynolds no.: 468
- Displacement th. Reynolds no.: 1135
- Skin friction coeff.: 1.834E-3
- Wall shear stress: 6.9E-2

---

**DISTANCE FROM LEADING EDGE = 1200 mm**

**SPANWISE LOCATION = 0**

**FREE STREAM VELOCITY = 10.51 m/s**

**AIR TEMPERATURE = 20 °C**

**ATMOSPHERIC PRESSURE = 765 mmHg**

<table>
<thead>
<tr>
<th>y</th>
<th>U w/s</th>
<th>y/d</th>
<th>u/ut</th>
<th>RMS</th>
<th>eta</th>
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</thead>
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<tr>
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<td>1.002</td>
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<td>5.117</td>
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</tbody>
</table>

RMS error of fit = 1.15616E-2

---

**Fig. 4.2.1** Printout from laminar analysis program
POLYNOMIAL FIT TO DATA

FILE 32/1200

\( \gamma = 0 \)

\( \delta = 4.85 \text{mm} \)

\( U_{\infty} = 10.51 \text{m/s} \)

Fig. 4.2.2  Polynomial fit to data
FILE PB/1208
DATA FOR TURBULENT BOUNDARY LAYER VELOCITY PROFILE

DISTANCE FROM L.E. - 1200
SPANWISE LOCATION - 0
FREESTREAM VELOCITY - 15.9
AIR TEMPERATURE - 21 Deg.C
ATMOSPHERIC PRESSURE - 753

Y= U/Unh RMS vel.
0 385 1.9E-2 .454 1.41
1.89 4.1E-2 .540 1.24
1.86 7.4E-2 .689 1.12
2.67 1.1E-1 .641 1.12
3.66 0.150 .669 1.09
4.46 0.160 .680 1.09
5.31 0.2 .714 1.07
6.22 0.234 .728 1.07
6.99 0.263 .744 1.06
8.42 0.324 .776 1.04
10.593 0.395 0.806 1.03
12.364 0.465 0.841 0.96
13.048 0.523 0.866 0.94
15.048 0.596 0.887 0.9
17.289 0.647 0.916 0.94
19.423 0.730 0.930 0.75
21.272 0.8 0.953 0.68
23.106 0.869 0.960 0.66
24.968 0.939 0.991 0.53
26.802 1.011 0.992 0.46
28.531 1.073 0.997 0.4
30.366 1.142 1.001 0.32
32.041 1.205 1.002 0.28

TURBULENT BOUNDARY LAYER INTEGRAL PARAMETERS

APPROX. EDGE OF BOUNDARY LAYER 26.59
DISPLACEMENT THICKNESS 4.747
MOMENTUM THICKNESS 3.332
ENERGY THICKNESS 5.025
SHAPE FACTOR H12 1.424
SHAPE FACTOR H32 1.748
MOM. TH. REYNOLDS No. 3475
DISP. TH. REYNOLDS No. 4950

CT (LUD/PILL) 2.994E-3
CT (LUD-PLOT) 2.942E-3
CT (COLES-FORM) 2.86E-3
WALL FRICTION VELOCITY 0.61
WALL SHEAR STRESS 0.442 N/m²
WAKE PARAMETER Pi 0.794

Fig. 4.3.3 Printout from turbulent analysis program
Fig. 4.3.4 Universal and Coles Composite turbulent boundary layer Velocity profiles
Favourable Pressure Gradient
\[ \frac{du}{dx} = 0.83 \frac{1}{s} \]

Zero Pressure Gradient

Adverse Pressure Gradient
\[ \frac{du}{dx} = -2.04 \frac{1}{s} \]

Fig. 4.8.1 Flow Two Dimensionality by the Momentum Balance Principle
5.1 Introduction

This chapter describes the development of a complete software package, related to the measurement and processing of boundary layer velocity profile data, for use with the BBC microcomputer. The function of the package is twofold; firstly to gather data from the experiments and dump this data to a disc file for permanent storage and secondly, to retrieve data from the disc files for subsequent analysis and display. The programs described in the previous two chapters for the data collection (Chapter 3) and data reduction (Chapter 4) from the basis of the package.

The package was developed specifically for use with a CUMANA double disc drive unit. Each drive of this unit has a double head enabling both sides of a disc to be used and, as each side of a disc has a storage capacity of approximately 200 k bytes, the facility makes available 800 k bytes of storage space. The top and underside of the lower disc and top and underside of the upper disc are numbered 0, 2, 1 and 3 respectively. The disc filing system (DFS) refers to the disc sides as drives ie drives 1 and 3 are sides 1 and 3 of the upper disc and drives 0 and 2 are sides 0 and 2 of the lower disc.

Drive Ø is called the BOOT drive as the BBC has an auto BOOT facility which allows a short introductory program, stored in the BOOT file, to be automatically loaded into the machine from drive Ø, and run by simultaneously depressing the SHIFT and BREAK keys.
Because of the limited amount of RAM available on the BBC micro the software package takes the form of a number of programs each dedicated to a specific task. The programs are interlinked using the BBC CHAIN command which enables one program to automatically load and run another program. The complete software package is stored on a master disc which is inserted into the lower drive of the disc drive unit leaving the upper drive, ie drives 1 and 3, free for the storage of raw data.

5.2 Running the Software Package

To augment the following description of the software package reference should be made to the flow diagram fig. [5.2.1]

The package is initiated by the auto boot facility (ie depressing the SHIFT & BREAK keys simultaneously) which immediately runs a BOOT program to CHAIN the program called PROGSEL. This program is used for the initial selection of whether:

1. an existing file is to be read; or
2. a new file is to be created.

**Option 1** If an existing file is to be read the name of the file and the drive number that the file is stored on are required and are fed in interactively from the keyboard. The name of the data file being read is then stored on disc, in a file called DATA. The name of the file can then be passed between programs by reading the file DATA at the beginning of each program. This streamlines the package as it eliminates the need for the file name to be typed in when a continuation program is CHAINed.

**Option 2** If a new file is to be created the data acquisition and control program described in Section 3.9 is called and a boundary layer traverse is initiated. On completion of the
traverse the raw data is dumped to a designated disc and the specified file name is stored in the file DATA as before.

A graphics program is then called which displays either the newly acquired data, or the data from a specified existing file, on axes of $y/\delta$ and $u/U_\infty$ along with the laminar Pohlhausen and turbulent $\nu/\rho$ power law velocity profiles for comparison. A hard copy of this display can be obtained by calling a screen dump program which causes the EPSON FX-80 printer to copy the graphics displayed on the RGB Monitor. (fig. [3.9.2] shows a screen dump printout of a graphical display.)

Once the data has been displayed a decision on which analysis is to be used for the reduction of the raw velocity profile data is required. There are three options available:

**Option 1 - laminar analysis** If this option is chosen the laminar analysis program described in section 4.2 is called and the reduced data is printed out on the RGB monitor. A hard copy can be obtained from the EPSOM printer, see fig. [4.2.1] No further analyses or displays are available from this stage.

**Option 2 - turbulent analysis** If this option is chosen the turbulent analysis program, described in section 4.3, is called and the reduced data is available as before (a printout of this reduced data is shown on fig. [4.3.2]). From this stage a plot of the data on the turbulent universal velocity profile can be obtained by dumping the Calculated $u^+, y^+$ data to a disc file, then calling the appropriate graphics program and retrieving the data from the file. A hard copy of this plot can be acquired by calling the screen dump program as described previously. No further analysis or displays are available from this stage.
Option 3 - transitional analysis  If this option is chosen a further decision on whether to use the laminar type analysis $\gamma \leq 0.5$ or the turbulent type analysis $\gamma \geq 0.5$ is required. The difference between the actual laminar and turbulent analysis described in sections 4.2 and 4.3 and the transitional versions of these programs is that the heading titles have been altered and the transitional skin friction value obtained from equation 4.21 is included in the analysis. The $\gamma(y)$ distribution is also printed out and the near wall intermittency value is calculated by averaging the $\gamma(y)$ values at all positions below $y/\delta = 0.2$, and is used for the solution of equation 4.21.

No further options are available and the program ends.

5.3 Special features of the package

After conducting an experiment, the data collected by the data acquisition and control program is dumped to a specified disc and the file is given a name. The data would normally be dumped to the upper disc, i.e. drive 1 or drive 3. If the specified drive is either full of data or the drive catalogue is full (the drive catalogue is capable of holding information on only 32 files) then the computer will sense an error code and terminate the program, thereby losing the collected data before it has been dumped to disc for permanent storage. To prevent this, the error code is intercepted within the software and the message, "Disk full-select another drive" is printed to screen. The data is then held in the computer memory until another drive has been specified or a new disc has been inserted into the drive unit. The program then down loads the data onto the new disc or alternative drive.
As all the package programs are stored on a Master disc, inserted into the lower drive, and the raw data files are stored on discs which are inserted into the upper drive, then switching between drives automatically within the software is necessary for smooth operation of the package. The drives can be selected within the software by the command *DR.X, but the X has to be an integer between 0 and 3 (eg *DR.1) and cannot be left as a free variable. For example the computer will not understand the following.

```
10 X = 3
20 *DR.X
```

To overcome this a small procedure was developed to enable a variable, say D, for example, to select a specified drive:-

```
10 DEFPROC Drive (D) - define procedure
20 IF D = 0 THEN *DR.0
30 IF D = 1 THEN *DR.1 select drive using value of variable D
40 IF D = 2 THEN *DR.2
50 IF D = 3 THEN *DR.3
60 ENDPROC - end procedure
```

Everytime a drive is to be selected by a variable the above procedure is simply called.

The drive number, on which the raw data file is stored, is passed between programs using one of the special variables A% to Z% on the BBC micro. These variables once specified are retained in the computer memory until they are overwritten or the computer is switched off. The BREAK or even CTRL/BREAK functions do not affect these variables.
To illustrate how data file names and drive numbers are passed between programs the following example, which can be considered as the end on one program and the start of a continuation program, is described.

Assume that the data file is stored on drive 3 and that the value D%, used to pass the drive number between programs, has been set to 3 at an earlier stage.

```
1000 PROC Drive (D%)
1010 Ch% = OPENOUT 'DATA'
1020 PRINT# Ch%, NAME$
1030 CLOSE# Ch%
1040 *DR.0
1050 CHAIN "Next prog"
```

```
10 REM next program
20 PROC Drive (D%)
30 Ch% = OPENIN 'DATA'
40 INPUT# Ch%, NAME$
50 Close# Ch%
```

1. select drive 3
2. open file DATA for output to disc
3. put character string in NAME$ to file 'DATA'
4. close file
5. load next program from drive 0 and RUN
6. select drive 3
7. open file for input to computer from file
8. read character string in NAMES from file DATA
9. close file
Fig. 5.2.1 Flow Diagram of Software Package
6.1 Introduction

With regard to the stated objectives a series of experimental flows were set up to investigate the influence of freestream turbulence and pressure gradient on the position and extent of the transition region in a boundary layer developing from the leading edge of a smooth flat plate. These flows are described in detail in this chapter and the results extracted from each flow are discussed and compared with alternative source data. The present method of defining the streamwise intermittency distribution through a transition region is compared to those of Abu-Ghannam & Shaw (1980) and Dhawan & Narasimha (1957) and statistical similarity of transition regions in zero and non-zero pressure gradients is observed. The transition length data acquired from the experiments is ultimately used to obtain a correlation which can be used to predict the combined effect of freestream turbulence and pressure gradient on the extent of the transition region.

6.2 Description of experimental flows

For each of the experimental flows the boundary layers were allowed to develop naturally from the leading edge of a smooth flat plate without the influence of external tripping devices such as vibrating ribbons, trip wires, surface roughness etc, to promote transition. The only factors considered to influence transition being the freestream turbulence level and the streamwise pressure distribution within the tunnel working section.
The natural test section turbulence level was approximately 0.35%. Three higher turbulence levels were generated by the insertion of various turbulence generating grids at a distance upstream from the leading edge, giving in all four test section freestream turbulence distributions. Details of these grids are given in section 2.4. For each grid the freestream turbulence distributions along the plate are remarkably constant as can be seen from fig. [2.4.1]. This is due to the positioning of the grids at a distance far upstream from the leading edge of the plate within the tunnel contraction, see fig. [2.1.1], allowing the grid generated turbulence to decay to an almost constant value before the flow reaches the plate leading edge.

Only the longitudinal component of the velocity fluctuation \( u' \) has been considered when calculating the freestream turbulence intensity. However, due to the constancy of the freestream turbulence distributions along the plate it is assumed that the turbulence is isotropic ie:-

\[
Tu = \frac{1}{3} \sqrt{u'^2 + v'^2 + w'^2} = \frac{\sqrt{u'^2}}{u_\infty} \times 100 \quad \ldots \ldots \quad 6.1
\]

No measurements of the \( v' \) and \( w' \) velocity components were made to justify this but Blair (1980) showed that by the time grid generated turbulence had decayed to an almost constant value the three constituent fluctuating velocity components \( u' \), \( v' \) and \( w' \) were approximately equal. Fig. [6.2.1] which is a reproduction of fig. (31) from Blair (1980) supports this premise.

Blair also observed, in accordance with Baines and Peterson (1951), that the turbulence levels generated for each grid
configuration were constant at a specific streamwise location, regardless of the freestream velocity. This agrees well with the freestream turbulence generated from the three grids used in this investigation as is shown in fig. [6.2.2]. A spectral analysis of the freestream turbulence levels generated by the various grids used for this investigation showed that in each case the bulk of the turbulence energy was contained at frequencies below 2 kHz. This is normal for grid generated turbulence, Meier & Kreplin (1980). The length scales of the freestream turbulence levels were also measured using a DISA-APA system and a similar set up to that used by Meier & Kreplin, to obtain auto-correlations for the turbulence generated by each grid. This showed that the streamwise length scales varied from 4 mm for grid 1 to 12 mm for grid 3. These values are similar to those obtained by Abu-Ghannam & Shaw (1980) who concluded that such a range of length scales would have a negligible effect on the position of the transition region. The conclusion from these measurements was that the various test section freestream turbulence levels generated for this investigation were concurrent with standard classical grid generated turbulence.

The pressure gradients were introduced by adjusting the variable height roof, as described in section 2.5, and are shown expressed in terms of pressure coefficient \( C_p \) in fig. [2.5.1]. The tunnel working section geometries were set up to give linear velocity distributions ie constant velocity gradients. Four roof settings were arranged to give, when expressed non-dimensionally, four velocity gradients ie a zero gradient \( \frac{d(U/U_0)}{d(X/L)} = 0 \), two
adverse gradients \( \frac{d(U/U_0)}{d(x/L)} = -0.23 \) and \( \frac{d(U/U_0)}{d(x/L)} = -0.15 \) and a favourable gradient \( \frac{d(U/U_0)}{d(x/L)} = 0.094 \). When expressed non-dimensionally in terms of \( \frac{d(U/U_0)}{d(x/L)} \) the velocity gradients are independent of the tunnel reference velocity and are approximately constant for each roof setting. (Slight variations in the velocity distributions can be detected when the flow reference velocity and freestream turbulence are altered. This is due to the varying rates of growth of the tunnel boundary layers but amount to a variation of less then 3% from the mean in the worst case.) The mean velocity distributions for each roof setting are plotted in terms of \( \frac{U_{\infty}}{U_0} \) and \( \frac{x}{L} \) in fig. [6.2.3].

In all, twenty three flows were investigated each having a different combination of freestream turbulence and pressure gradient. Details of each of these flows are given in table 6.1 and are described briefly below.

Flows 1-4 are zero pressure gradient flows with freestream turbulence levels ranging from 0.35% to 1.40%. In order to position the transition region within the working area of the plate these flows had to be tested at the maximum tunnel reference velocity of, nominally, 18 m/s. Even at this maximum tunnel reference velocity the measured end of transition for the two lower freestream turbulence flows, ie flows 1 and 2, occurs just beyond the safe working area of the plate and in consequence are thought to occur prematurely in both cases.
Flows 5 + 12 are adverse pressure gradient flows. The working section geometry for these flows were set for the first non-dimensional adverse velocity gradient. Flows 5 to 8 are tested at the maximum tunnel reference velocity of 18 m/s while flows 9 to 12 are tested at a reduced reference velocity of nominally 10 m/s, giving two velocity gradients ie $\frac{du}{dx} = -2.2s^{-1}$ and $\frac{du}{dx} = -1.2s^{-1}$. Each flow was a different combination of velocity gradient and freestream turbulence. The relevant areas of interest for each of these flows occurs well within the safe working area of the plate.

Flows 13 to 20 are also adverse pressure gradient flows but with the working section geometry altered to give the second non-dimensional adverse pressure gradient. These flows were tested, as before, at tunnel reference velocities of nominally 18 m/s and 10 m/s giving two further velocity gradients $\frac{du}{dx} = -1.4s^{-1}$ and $\frac{du}{dx} = -0.75s^{-1}$.

Flows 21 to 23 are favourable pressure gradient flows. Unfortunately because of the tunnel geometrical constraints the adjustable roof had to be lifted above the level of the contraction outlet, in the region of the plate leading edge, to obtain a favourable pressure gradient within the tunnel working section, see fig. [6.2.4] below.

![Fig. 6.2.4 Cross-section through tunnel](image-url)

*Fig. 6.2.4 Cross-section through tunnel Showing roof setting for favourable pressure gradients*
Consequently as the flow enters the working section it is subjected to a fairly strong adverse pressure gradient, before it impinges on the plate leading edge and over the initial 150 mm of the plate surface. Due to the effect of this initial adverse pressure gradient the high freestream turbulence level flow (ie the flow with Grid 3 in position - Tu 1.4%), becomes transitional within the first 100 mm of the plate surface. This is far earlier than would have been expected had it been possible to set up a favourable pressure gradient over the entire length of the plate. For this reason no measurements are presented for grid 3 flows for the favourable pressure gradient case. Transition for flows 21, 22 and 23 begins well downstream of the initial adverse pressure gradient and disturbances within the boundary layer amplified by the initial adverse pressure gradient (though not enough to promote transition) are likely to have been damped out by the calming influence of the favourable pressure gradient, Schubauer & Skramstad (1948). However it would be reasonable to expect the initial adverse pressure gradient to have some detrimental effect on the boundary layer which is likely to promote early transition. The favourable pressure gradient flows described above were tested at the tunnel maximum reference velocity of 18 m/s to ensure that, at least the start of transition would be positioned within the plate safe working area. Disappointingly only in flow 21 (Tu 0.8%) did the end of transition occur within this region. The end of transition for flows 22 and 23 occurs well outwith the safe working area and as a consequence it is likely that the length of the transition region is shorter than would normally be expected.
Bearing in mind the observations overleaf the data available from the favourable pressure gradient flows, with the present experimental facility, is very limited.

6.3 Flow Measurements

For each of the twenty three flows described a series of measurements were made; these were:-

(i) Hot wire boundary layer traverses at regular streamwise intervals, along the centre line of the plate, measuring the mean velocity, rms velocity, and intermittency distribution through the boundary layer perpendicular to the plate.

(ii) The streamwise intermittency distribution along the plate centreline.

(iii) The streamwise freestream turbulence distribution.

(iv) The streamwise freestream velocity distribution.

The position of the start and end of transition were also noted and the velocity profiles at or very close to these positions were measured for each flow. This enabled measured values of integral thicknesses to be obtained at the start and end of transition.

A summary of the relevant data extracted from each flow is given in table 6.2. The freestream turbulence levels given in this table are the values at the position of start of transition ie

\[ T_{us} = \frac{\sqrt{u'^{2}_s}}{U_{\infty s}} \times 100 \]  

.......... 6.2
although they vary very little, if at all, from the nominal values given in table 6.1 measured at the position of the plate leading edge.

A limited number of measurements from various other flows supplementary to the twenty three flows described in section 6.2 were made. These flows were tested at low tunnel reference velocities ($U_0 = 5$ m/s for the adverse gradients and $U_0 = 10$ m/s for the favourable gradient) and the only data extracted from them was the position of the start of transition. For this reason these flows were not numbered but when data from such a supplementary flow is presented on a diagram the relevant parameters are given alongside the plotted point.

6.4 Description of the transition process

Since the experiments of Schubauer & Skramstad (1948) it has generally been accepted that the breakdown process from laminar to turbulent motion, within a boundary layer, involves the amplification of small two dimensional disturbances superimposed on the laminar flow (ie Tollmein-Schlichting waves). At some critical Reynolds number the Tollmein Schlichting waves become unstable and grow as they move downstream eventually breaking down into bursts of random fluctuation characteristic of turbulent flow. These bursts occur in small localised regions in the form of turbulent spots, first observed by Emmons (1950), and grow in size as they travel downstream until they coalesce into a fully developed turbulent boundary layer. The region between the first occurrence of these turbulent spots and the position at which they merge to form a fully turbulent boundary layer is termed
the transition region. Arnal et al (1977), Schubauer & Klebanoff (1956), Emmons (1950) and many others since have observed that the flow within a transition region alternates between the laminar and turbulent flow states. Although separate laminar and turbulent velocity components were not measured in this investigation, evidence to support this can be gleaned from measurements of the instantaneous velocity within a transitional boundary layer. Fig. [6.4.1] shows oscilloscope traces of the instantaneous velocities measured by a hot wire probe placed both near to the wall, and in the outer region of a transitional boundary layer. With the probe placed close to the wall (Trace (a)) the signal shows an increase in mean level as a turbulent region is encountered, and with the probe placed in the outer region of the boundary layer (Trace (b)) the signal shows a decrease in mean level as a turbulent region is encountered.

These observations are consistent with the sketch of the mean laminar and turbulent velocity profiles shown in this figure. At \( y < y_c \) the turbulent component of mean velocity is greater than the laminar component and at \( y > y_c \) the laminar component of the mean velocity is greater than the turbulent component. The implication here is that the transitional boundary layer consists of laminar and turbulent mean velocity components qualitatively similar to those shown in fig. [6.4.1].

Confirmation of this physical model is given by both Wygnanski et al (1976) and Arnal et al (1977). These researchers used conditional sampling techniques to measure the constituent profiles in and out of turbulent spots and verified that the component profiles within a transition region were characteristic of mean laminar and mean turbulent velocity profiles.
This model of the transition region can be used to describe the continuous growth of the mean boundary layer properties, through transition, from typical laminar to typical turbulent values: The process of transition involves an increase in momentum, displacement and energy thicknesses; a decrease in the displacement thickness shape factor $H_{12}$ and a slight, but significant, increase in energy thickness shape factor, $H_{32}$. The skin friction coefficient also increases from a laminar to a turbulent value over the length of the transition region.

Typical distributions of the boundary layer parameters through a transition region are shown in fig. [6.4.2]. The points plotted on this figure are experimental points and the lines are mean lines drawn through the experimental data.

6.5 **Start of transition - Correlations**

The point at which a laminar boundary layer first becomes unstable (point of instability) is normally expected to lie upstream of the experimentally observed point of transition (ie the point at which turbulent bursts first appear) and the distance between these two points is dependent on the degree of amplification of disturbances within the boundary layer and the type of disturbance present in the freestream. Although a fair amount of success has been adhered in predicting the influence of various effects on the limit of stability, Schlichting (1979); to date no rational explanation from first principles is available for predicting the onset of turbulence, ie the transition point.

In order to obtain a practical solution to the prediction of the transition point, researchers have been striving to
obtain realistic correlations which are applicable to as wide
a range of practical situations as possible. To achieve this
the factors which influence the position of transition must be
identified and the major effects incorporated into the correlation.
Two such effects which have been correlated with some success are the
influence of freestream turbulence and pressure gradient on the
transition point. In the absence of a pressure gradient it is
known that increasing the freestream turbulence level will advance
the onset of transition. This effect has been fairly well
correlated by a number of researchers; two such correlations due
to Van Driest and Blumer (1963) and Hall and Gibbings (1972) for
zero pressure gradient flows are shown in fig. [6.5.1]. Plotted
on this figure is the present data from the zero pressure
gradient flows (FLOWS 1 - 4) which compare favourably with the
correlations. Both correlations are similar and show that the
effect of freestream turbulence on the position of transition
(defined in terms of $R_{qg}$ in fig. [6.5.1]) asymptotes to an almost
constant value as $Tu$ increases, the bulk of the effect being
contained within the region $0 \leq Tu \leq 2\%$. A more recent correlation
by Abu-Ghannam and Shaw (1980) is also shown on fig. [6.5.1]
represented by a chain dotted line. This correlation is almost
identical to that of Hall and Gibbings except that it has been
modified to asymptote to the Tollmein-Schlichting stability limit
$R_\theta = 163$, rather than the value of $R_\theta = 190$ used by Hall and Gibbings,
to fit the wider range of data supplied by Brown and Burton (1978)
and Martin et al (1978) for freestream turbulence levels up to 9.2\%.
The correlations of Abu-Ghannam & Shaw, Van Driest & Blumer and others such as those due to Dunham (1972) and Seyb (1972) were developed further to take into account the combined effect of freestream turbulence and pressure gradient on the location of the transition point. It seems generally agreed that the effect of pressure gradient can be best correlated against $R_{0g}$ and $Tu$, when expressed in terms of a non-dimensional pressure gradient parameter, the most common being the Thwaites (1949), or modified pohlriausen parameter $\lambda_0 = \frac{c^2}{\nu} \frac{dU_\infty}{dx}$; although recently the acceleration parameter $K = \frac{v}{U_\infty} \frac{dU_\infty}{dx}$ has become popular for quantifying the strength of favourable pressure gradients. See Brown & Martin (1976) and Blair (1982). The merits of these two parameters will be discussed in a later section.

Data from the present study is shown plotted against the most recent of these correlations, ie that of Abu-Ghannam and Shaw, on fig. [6.5.2]. For adverse pressure gradients, ie $\lambda_0 < 0$ all the correlations mentioned previously shown the same general trend but for favourable pressure gradient $\lambda_0 > 0$ the correlations of Dunham (shown as a chain dotted line in fig. [6.5.2]) and Seyb are in marked disagreement with the Abu-Ghannam & Shaw correlation, both showing a rapid increase in $R_{0g}$ with increasing $\lambda_0$.

The limited amount of favourable pressure gradient data from the present study and that of Blair (1982) would appear to substantiate the correlation of Abu-Ghannam & Shaw.
The value of $R_{gs}$ gathered from the adverse pressure gradient flows in the present investigation generally lie below those forecast by Abu-Ghannam & Shaw's correlation. Two reasons for this are:

(i) The method of detecting transition in the present work locates the start of transition very close to the appearance of the first turbulent bursts which is likely to be upstream of the location identified by Abu-Ghannam & Shaw because of the different detection techniques used (see section 4.7 for more detail). For this reason values of $R_{gs}$ obtained by Abu-Ghannam & Shaw are likely to be higher than those obtained here for similar flow conditions.

(ii) The adverse pressure gradients were arranged to be effective directly from the plate leading edge, therefore, any leading edge disturbances would be amplified by the amplifying effect of the adverse pressure gradient and may lead to a breakdown of the laminar boundary layer at values of $R_{gs}$ less than would normally be expected. In retrospect it may have been advantageous to arrange a fairly strong favourable pressure gradient in the vicinity of the plate leading edge (say over the first 50 mm, or so, of the plate surface). This would have had the beneficial effect of damping any leading edge disturbances.

The correlation proposed by Abu-Ghannam & Shaw is based on a wide range of experimental data so there would be no justification to tune the correlation to give a better fit to the present data. However, having identified the above effects the present data is fairly reasonable and when compared to the actual data of Abu-Ghannam & Shaw it is within acceptable experimental scatter.
6.6 Statistical similarity of transition regions

As previously described in section 6.4 the transition region is composed of intermittent regions of laminar and turbulent flow. The turbulence originating in the form of small spots which grow downstream and eventually coalesce to form a completely turbulent flow regime.

Emmons (1951), who pioneered the concept of turbulent spots, introduced a spot source density function to describe the production of these spots and related this to the probability of the flow being turbulent at any time, ie the intermittency function $\gamma$. Dhawan & Narasimha (1957) argued that the spot source density function should have a maximum value at some point, which can be considered as the experimentally measured point of transition, and that downstream of this point the turbulence probability could be defined by the unique relation

$$\gamma = 1 - e^{-A^2 \xi^2} \quad \ldots \quad 6.3$$

where $\xi = (x - x_s)/\lambda$ is a normalised streamwise co-ordinate in the transition region and $\lambda$ is a measure of the extent of the transition region.

Dhawan & Narasimha showed that the distribution of intermittency was universal on the $\bar{\gamma}(\xi)$ plot irrespective of the physical length of the transition region. This was supported by the earlier work of Schubauer & Kelbanoff (1956) who gave rise to the concept of the statistical similarity of transition regions. In contrast to Dhawan & Narasimha, Schubauer & Klebanoff normalised the streamwise intermittency distribution ($\bar{\gamma}$) to the normal distribution function, matching the curve at $\bar{\gamma} = 0.5$ using the normalised streamwise co-ordinate $\xi = (x - \bar{x})/\sigma$. Where $\sigma$, the standard deviation, is a measure of the extent of the transition region and $\bar{x}$ is the distance from the leading edge to the point where $\bar{\gamma} = 0.5$. 

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In both papers mentioned overleaf, the intermittency distributions were measured only in zero pressure gradients and both make the observation that it does not necessarily follow that transition regions will exhibit the same distribution of $\bar{\gamma}$ in the presence of a pressure gradient.

The present investigation makes use of the work by Schubauer & Klebanoff, in assuming the value $\sigma$ to be representative of the transition length. A computer program was written to calculate the mean value of $\sigma$ from the experimentally measured $\bar{\gamma}(x)$ distribution stored in a disc file. For computational convenience the normal distribution or Gaussian integral curve was represented by a polynomial approximation to the curve, i.e., equation 6.4.

$$\bar{\gamma} = \frac{1}{\sqrt{\pi}} \left[ 1 + \frac{\zeta}{|\zeta|} (0.8273 |\zeta| - 0.096 |\zeta|^2 - 0.073 |\zeta|^3 + 0.0165 |\zeta|^4) \right]$$

where $\zeta = \frac{x - \bar{x}}{\sigma}$

(Care has to be taken of the singularity point at $\zeta = 0$)

The polynomial approximation to the normal distribution is shown on fig. [6.6.1]. To the scale of this diagram no difference between the two is detectable.

The procedure for determining the standard deviation of the intermittency distribution was to firstly obtain the value of $\bar{x}$, the location of the $\bar{\gamma} = 0.5$ point. This was done by fitting a least squares straight line to all the experimentally measured points in the region $0.75 < \bar{\gamma} > 0.25$, as the distribution is approximately linear in this region, and determining $\bar{x}$ from the resulting straight line equation. Using the experimentally obtained values of $\bar{\gamma}$ for each point the value of $\zeta$ is determined.
by iteration of equation 6.4 and substitution of this value into equation 6.5 gives the value of \( \sigma \) for each point. The average value of \( \sigma \) is then taken to be representative of the complete distribution. The experimental \( \overline{\gamma} (x) \) data is then converted to \( \overline{\gamma} (\zeta) \) data using the calculated values of \( \overline{x} \) and average \( \sigma \) value. These experimental points are then plotted graphically on the display monitor and compared to the normal distribution represented by equation 6.4.

Assuming \( x_S \) is measured at \( \gamma = 0.01 \) and \( x_e \) is measured at \( \gamma = 0.99 \) then the corresponding normalised co-ordinate \( (\zeta) \) for the start and end of transition will be -2.30 and +2.30 respectively. The length of the transition region can therefore be related to the standard deviation \( \sigma \) by the relation

\[
x_\zeta = 4.6 \sigma
\]

or non-dimensionally

\[
Rx_\zeta = 4.6 R \sigma
\] ......... 6.6

The experimentally measured value of \( x_\zeta = (x_S - x_e) \) is plotted against the value of \( x_\zeta \) calculated from equation 6.6 in fig. [6.6.2]. Excellent agreement between the measured value of \( x_\zeta \) and that obtained from equation 6.6 can be seen from this figure.

Fig. [6.6.3 (a)] shows the intermittency data from the present experimental flows plotted separately for the adverse, zero and favourable pressure gradient cases, against the \( \gamma (\zeta) \) distribution. Figs. [6.6.3 (b)] and [6.6.3 (c)] show the same data plotted in the \( \gamma (\xi) \) distribution of Dhawan & Narasimha and the \( \gamma (\eta) \) distribution of Abu-Ghannam & Shaw. It can be
seen from these figures that the present method of defining the intermittency distribution is superior to the other two methods mentioned overleaf. The reason for this lack of agreement between the present distribution, $\gamma (\xi)$ and the $\gamma (\xi)$ and $\gamma (\eta)$ distributions of Dhawan & Narasimha and Abu-Ghannam & Shaw respectively, is that the latter two distributions use $x_s$ as the datum length, whereas $\bar{x}$, used in the present method is more readily defined. Also the parameters used for normalising the transition co-ordinates $\xi$ and $\eta$, by Dhawan & Narasimha and Abu-Ghannam & Shaw are defined by only two points in the transition region. The normalising parameter, $\sigma$, used in the present method is calculated for each data point and an average value is used to define the distribution.

From fig. [6.6.3 (a)] and fig. [6.6.2], it can be seen that neither the pressure gradient nor the freestream turbulence has any influence in the intermittency distribution when expressed in terms of $\gamma$ and $\xi$. This is consistent with the observations of Abu-Ghannam & Shaw. However, it is likely that if the pressure gradient was to alter drastically within the transition region (e.g. if it were to change sign) then the intermittency distribution expressed in terms of $\gamma (\xi)$ might not follow the normal distribution curve shown in fig. [6.6.1].

6.7 The effect of freestream turbulence on transition length (zero pressure gradient)

As described in the previous sections the degeneration of the flow from the laminar to the turbulent state is not instantaneous but occurs over a finite length. Although a number of researchers have conducted experiments to determine the influence of various
effects on the point at which this degeneration begins, see
section 6.5, very little information is available on the
influence of the various parameters on the extent of the transition
region.

A popular method of obtaining the transition length is to use
the very approximate relationship of Dhawan & Narasimha who
defined the Reynolds number, based on the transition length
parameter \( \lambda \), as a function of the length Reynolds number at the
start of transition, equation 6.7.

\[
R_\lambda = 5 R_x^{0.8} 
\]

\[ \cdots \cdots \cdots \text{6.7} \]

From the data presented by Dhawan & Narasimha, Dunham (1972)
observed that the total transition length, \( x_L \), could be related
to the length parameter \( \lambda \) by the relationship

\[
x_L = 3.36 \lambda
\]

\[ \cdots \cdots \cdots \text{6.8} \]

Therefore, modifying Dhawan & Narasimha's original relationship
to give the transition length, results in:-

\[
R_{x_L} = 16.8 R_x^{0.8}
\]

\[ \cdots \cdots \cdots \text{6.9} \]

Dunham further modified this using the Blasius relation

\[
R_\theta = 0.694 \sqrt{R_x}, \text{to}
\]

\[
R_{x_L} = 31.8 R_\theta^{1.6}
\]

\[ \cdots \cdots \cdots \text{6.10} \]

for zero pressure gradient flows. Debruge (1970) proposed
a similar correlation to that of Dhawan & Narasimha, merely
adjusting the constant and power terms to fit his particular
range of data.

\[
R_\lambda = 0.005 R_x^{1.28}
\]
Even as recently as 1980, Abu-Ghannam & Shaw used an unmodified version of Dhawan & Narasimha's original correlation to determine the extent of the transition region and used this as the basis for calculating the boundary layer development through the transition region.

However, Dhawan & Narasimha stated originally that their proposed correlation was no more than speculative due to the considerable degree of scatter of the experimental data (disguised by a log-log plot, in Fig. 5 of Dhawan & Narasimha's original paper). They also suggested that a family of $R_\lambda v's R_{\infty}$ curves, each depending on the specific agency causing transition, would be more realistic.

The present approach to defining the transition length is somewhat different to the Dhawan & Narasimha approach in that the effects influencing the transition length are directly correlated to a transition length Reynolds number based on the length parameter $\sigma$, i.e.

$$R_\sigma = \frac{U_{\infty} \sigma}{v} \quad \ldots \ldots \quad 6.11$$

(The length parameter $\lambda$ can be related to $\sigma$ by the relationship $\lambda = 1.37\sigma$). This new approach is however, consistent with the implications of Dhawan & Narasimha's relationship i.e. equation 6.7.

The present correlation for the effect of freestream turbulence on the transition length in zero pressure gradient flow is shown on Fig. [6.7.1]. ($R_\sigma$ is correlated to the local value of freestream turbulence at the transition point although as described earlier the freestream turbulence level is almost constant over the entire length of the plate). Also plotted
on this figure is the present data and that of previous workers. Unfortunately transition length/freestream turbulence level data is very scarce for zero pressure gradient flows and is almost non-existent for non-zero pressure gradient flows, in the presently available literature.

The available data is best correlated by:

\[
R_g = \left[ 270 - \frac{250 \, T_u^{3.5}}{(1 + T_u^{3.5})} \right] \times 10^3 \quad \ldots \ldots \quad 6.12
\]

The upper limit for \( R_g \) was obtained from Schubauer & Skramstad (1947). Schubauer & Skramstad showed that \( R_{xg} \) reached an upper limit at a freestream turbulence level of about 0.1% and decreasing the freestream turbulence below this value had no further effect on \( R_{xg} \). The value of \( R_{xg} \) was also constant in this range but did increase from \( 0.1 < T_u < 0.25 \). However, flows with freestream turbulence levels of less than 0.25% are probably of limited practical significance, excepting the case of free flight, so the value of \( R_g = 270 \times 10^3 \) obtained from Schubauer & Skramstad, for flows with \( T_u < 0.1 \), was taken to be the upper limit and is held constant at this value until \( T_u = 0.25 \) is exceeded. At this point there is a rapid decrease in \( R_g \) with increasing freestream turbulence, eventually asymptoting to a lower limiting value, as implied by Abu-Ghannam & Shaw and Hall & Gibbings, of \( R_g = 20 \times 10^3 \). This concept of a lower limit is thought to be reasonable as transition is always expected to occur over some finite length.
6.8 Combined effect of freestream turbulence and adverse pressure gradient on transition length

Whereas it is possible to eliminate the effect of freestream pressure distribution enabling the independent effect of freestream turbulence to be examined, it is not possible, due to the natural level of freestream turbulence present in all wind tunnels, to separate the effect of freestream turbulence from pressure distribution in non-zero pressure gradient flows. In consequence only the combined influence of the two effects on the transition length can really be examined. However, for the present investigation the pressure gradients were all arranged to give constant velocity gradients and the freestream turbulence levels attributed to each flow were nominally constant over the plate working length. Each flow could therefore be specified by a single value of velocity gradient, \( \frac{dU_\infty}{dx} \), and freestream turbulence level, \( T_u \). It was thus possible to compare the effect of varying \( \frac{dU_\infty}{dx} \) on the transition length and location for a range of constant freestream turbulence flows and also to compare the effect of varying \( T_u \) on the transition length and location for a range of constant velocity gradient flows. Figs. [6.8.1], [6.8.2] and [6.8.3] were constructed, therefore to give an indication of the separate effects. (The dotted lines drawn through the data on these figures serve to highlight the effects and are not meant to imply any specific relationship).

From fig. [6.8.1] it can be seen that the effect of increasing the velocity gradient at a constant value of freestream turbulence has the effect of advancing the onset of transition and, to a greater extent, advancing the position at which transition ends, ie the position where the flow becomes fully
turbulent. Hence increasing $\frac{\partial u}{\partial x}$ has the effect of decreasing the transition length. This substantiates the observations of Schubauer & Klebanoff (1956) and Tani (1969). From this figure it can also be seen that the influence of increasing the velocity gradient (ie the adverse gradient becoming more negative) on the position of the start of transition is less significant as the freestream turbulence level is increased.

Fig. [6.8.2] shows the effect of increasing freestream turbulence level in a constant velocity gradient flow. As can be seen from this figure the effect of increasing freestream turbulence appears to increase the transition length. This effect is most significant for low freestream turbulence levels, below 1%, and tends to fade for higher turbulence levels and may in fact reverse to give a decrease in transition length with further increase in freestream turbulence level.

This can be explained by examination of fig. [6.8.3] as follows: At low freestream turbulence levels approximately below 1% slight increases in the value of freestream turbulence have a marked effect in advancing the onset of transition but appear to have less of an effect on the end of transition. This is due to the fact that the end of transition has already been advanced to, perhaps, a more stable position by the effect of the adverse velocity gradient and is not likely to be advanced further by low freestream turbulence levels or small changes in freestream turbulence. However at higher freestream turbulence levels ($Tu \geq 1.0$) the effect on the advancement of the transition point by increasing the freestream turbulence level becomes less significant until, as in the case of zero pressure
gradient flows, further increase in Tu causes no further advancement of the start of transition. As this asymptotic position is approached the lengthening effect of increasing Tu will decrease and may in fact reverse if the rate at which the end of transition is advancing due to increasing Tu is greater than that of the start of transition. At some point both the position of the start and end of transition will reach their respective minimum limiting values where further increase in Tu will result in no further effect on transition length.

6.9 The effect of favourable pressure gradient on transition length

Although a considerable amount of experimental effort was expended in setting up and making measurements in favourable pressure gradient flows, the effect of the velocity gradient on transition length can only be examined for one single flow (ie Flow 21). The other favourable velocity gradient flows (Flows 22 & 23 and various other supplementary flows) are either severely affected by the adverse pressure gradient in the region of the plate leading edge, see section 6.2, or the end of transition occurs beyond the safe working region of the plate. However from flow 21 and the zero pressure gradient counterpart, flow 3, (both with a freestream turbulence level of 0.8%), it can be seen that the effect of introducing a favourable velocity gradient is to delay the start of transition and to a greater extent delay the end of transition, hence increasing the transition length. This effect is shown in Fig. [6.9.1] and as would be expected is opposite to the effect observed in adverse velocity gradient flows. Unfortunately no
comments can be made as to the effect of freestream turbulence on the transition length in favourable velocity gradient flows as there is insufficient data.

6.10 Correlating the combined influence of freestream turbulence and pressure gradient on transition length

As described in the previous section there is only one favourable pressure gradient flow (Flow 21) for which transition length data can be confidently extracted. For this reason the correlation presented here is limited to zero and adverse gradient cases but could possibly be modified to account for favourable gradients if reliable data becomes available.

The major obstacle in correlating experimental data is in defining adequate non-dimensional parameters which are sufficient in independent variables to describe the problem. The present transition length data and that of previous researchers appears to correlate fairly well in terms of the transition length Reynolds Number $R_g$, and the local value of freestream turbulence at the start of transition $T_{us}$, as shown in fig. [6.7.1]. To correlate the transition length data in adverse pressure gradients the present approach was to modify this correlation using some parameter, involving the velocity gradient, which would describe the effects outlined in section 6.8. This approach has been used in the past by other researchers to correlate the position of the onset of transition in non-zero pressure gradient flows. The most popular parameter to account for pressure gradient effects being the modified Pohlhausen/Thwaites parameter, $\lambda_0 = \frac{g^2}{v} \frac{dU_0}{dx}$,
although other parameters such as the acceleration parameter
\[ K = \frac{\nu}{U_\infty^2} \frac{dU_\infty}{dx} \]
and a non-dimensional velocity gradient parameter
\[ \frac{d}{dx} \frac{U_\infty}{U_0} \frac{x}{L} \]
have been used by other researchers such as Brown & Burton (1978) and Blair (1982).

A fairly recent paper by Brown & Burton (1976) and discussion by Gibbings and Slanciauskas & Pédissius, reviews the merits of the modified Pohlhausen/Thwaites parameter, \( \lambda_0 \) and the acceleration parameter, \( K \). Brown & Burton suggest that \( K \) is a more suitable parameter than \( \lambda_0 \) for correlating pressure gradient effects mainly because it is composed of independent variables which are directly measurable and therefore more readily usable by the design engineer. This is obviously an advantage, but a distinct disadvantage of this parameter is that it does not, through any of its component variables, account for the history of the flow. The present author suggests that \( \lambda_0 \) is a more suitable parameter for correlating the effects of freestream turbulence and pressure gradient on the position and extent of transition for the following reasons:

(a) to some degree the history of the flow is taken into account through the inclusion of the boundary layer momentum thickness as a variable;

(b) when using a local value of \( \lambda_0 \) at the start of transition or perhaps an averaged value of \( \lambda_0 \) as suggested by Granville (1953) and Abu-Ghannam & Shaw (1980), the parameter is influenced by both the freestream turbulence and velocity gradient;
(c) more important is the fact that the present transition length data appears to correlate well in terms of $R_a$, $T_u$ and $\lambda_\theta$.

The correlation presented on fig. [6.10.1] uses the local value $\lambda_{\theta_s}$ although an averaged value of $\lambda_\theta$ from the origin of the boundary layer to the point of transition would account further for previous flow history. However with the linear velocity gradients used in this investigation the $\lambda_\theta$ distribution is almost linear, therefore such an average would merely result in halving the local values at the start of transition. Abu-Ghannam & Shaw actually found no improvement in their correlation by using a mean value of $\lambda_\theta$ defined by

$$\lambda_\theta = \frac{1}{x_s - x_0} \int_{x_0}^{x_s} \lambda_\theta \, dx \quad \ldots \ldots \ldots \ldots \quad 6.13$$

but did in fact find an improvement in correlation when using the extreme value of $\lambda_\theta$ ie the local maximum value of $\lambda_\theta$. For the present experimental flows $\lambda_\theta$ (extreme) will always occur at the start of transition ie $\lambda_{\theta\text{ (extreme)}} = \lambda_{\theta_s}$.

It may seem rather speculative to relate the transition length to local values at the start of transition but the degree of correlation would appear to justify this speculation.

The final correlation shown in fig. [6.10.1] is represented by

$$R_a = \left[ 270 - \frac{250 \, T_u^{3.5}}{1 + T_u^{3.5}} \right] \left[ \frac{1}{1 + 1710(-\lambda_\theta)^{1.4} \exp^{-\sqrt{1+T_u^{3.5}}}} \right] \times 10^3 \quad 0 < \lambda_\theta < -0.04 \quad \ldots \ldots \ldots \ldots \quad 6.14$$

$$R_a = 20 \times 10^3 \quad \lambda_\theta < -0.04$$
It may appear that this correlation does not illustrate the
effect described in section 6.8, that at low freestream
turbulence levels increasing $T_u$ increases the transition length.
However as suggested previously in this section the parameter $\lambda_\theta$
accounts for the combined freestream turbulence and pressure
gradient effect. Increasing freestream turbulence advances the
onset of transition and hence reduces $\lambda_\theta$, which is indicated
in the correlation by an increase in $R_\theta$ at low values of $T_u$.
At higher freestream turbulence levels (say above 1%) the
effect of increasing freestream turbulence on $\theta_s$ is small, however
$\lambda_\theta$ will still reduce but not to the extent that $R_\theta$ increases, as
at the higher levels of freestream turbulence the direct effect
of $T_u$ is having a dominant effect in reducing $R_\theta$.

The limit of $R_\theta = 20 \times 10^3$ at $\lambda_\theta < -0.04$ was specified to
fit the present adverse pressure gradient data and to correspond
to the limit in the zero pressure gradient correlation of $R_\theta$
and $T_u$.

The experimentally measured values of $R_\theta$ are plotted
against those obtained from equation 6.14 on Fig. [6.10.2].
Fig. [6.10.3] shows the experimentally measured values of $R_\lambda$
plotted against those obtained from the Dhawan & Narasimha
correlation (equation 6.7). Comparison of these two figures
shows the marked improvement of the present correlation over
that of Dhawan & Narasimha for the present data. Unfortunately,
very little alternative source data is available in the present
literature which can be plotted on this correlation. It is
hoped, however, that this work will show other researchers
that it is possible to correlate the transition length, expressed
non-dimensionally as a transition length Reynolds number, directly in terms of external influences such as freestream turbulence and pressure gradient. Also it is hoped that this will stimulate other researchers to producing a wider range of suitable data and that the correlation can then be tuned or modified to suit a wider range of practical situations.
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<th>$U_o$ (m/s)</th>
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<th>$T_u$% (Nominal)</th>
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<td>0.90</td>
<td>0.40</td>
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Table 6.1 Test Flow details
Table 6.2 Results Summary

*(a) Calculated from measured velocity profile
(b) predicted by Tani's (1954) method
Fig. 6.2.1 Streamwise distribution of freestream turbulence components (reproduction from Blair (1980))
Fig. 6.2.2. Turbulence intensity as a function of velocity.
Fig. 6.2.3. Mean freestream velocity distributions for the four working section geometries tested.

$\frac{u_\infty}{u_0}$ vs $x/L$ ($L = 2000$)
The above traces were traced from photographs of a storage oscilloscope screen.

Fig. 6.4.1 Instantaneous velocity within a transitional boundary layer ($\gamma \approx 0.1$)
Fig. 6.4.2 Development of boundary layer through transition
1. - Van-Driest & Blumer  \( R_{\theta_S} = -29.52 R_{\theta_S}^2 Tu^2 + 122 \)
2. - Hall & Gibbings  \( R_{\theta_S} = 190 + \exp(6.88 - 1.03 Tu) \)
3. - Abu-Ghannam & Shaw  \( R_{\theta_S} = 163 + \exp(6.91 + Tu) \)
4. - Abu-Ghannam & Shaw  \( R_{\theta_e} = 2.667 R_{\theta_S} \)

* - present data

Fig. 6.5.1 Start of transition correlations (zero pressure gradient)
Fig. 6.5.2. Abu-Ghannam & Shaw correlation for $R_{0S}$ in non-zero pressure gradient flows
Fig. 6.6.1. Polynomial approximation to normal distribution
Fig. 6.6.2. Plot of $X_1$ against $\sigma$

$X_1 = 4.6\sigma$
Fig. 6.6.3. (a) present $\gamma$ vs $\zeta$ Intermittency distribution
Fig. 6.6.3. (b) Dhawan & Narasimha Intermittency Distribution

\[ \xi = \frac{X - X_0}{\lambda} \]
$\eta = \frac{X - X_S}{X_e - X_S}$

Fig. 6.6.3. (c) Abu-Ghannam & Shaw Intermittency Distribution
Fig. 6.7.1. Present zero pressure gradient correlation
Fig. 6.8.1. Velocity gradient effect on transition length ($U_0=18\text{m/s}$ for all cases)
Fig. 6.8.2. Effect of freestream turbulence on transition length for constant velocity gradients.
Fig. 6.8.3. Effect of freestream turbulence on start and end of transition for constant velocity gradient
Fig. 6.9.1. Effect of fav velocity gradient on transition length

\[
\frac{dU_\infty}{dx}
\]

- start of transition
- end of transition
Fig. 6.10.1. Present correlation - combined effect of freestream turbulence and pressure gradient on transition length
Fig. 6.10.2 $R_g$ (equation 6.14) against $R_g$ (measured)
Fig. 6.10.3 Present Data Compared with Dhawan & Narasimhas' Production
CHAPTER SEVEN

Prediction of the Transition Boundary layer development

7.1 Introduction

Early methods for predicting the development of a boundary layer over a surface frequently ignored the transition region by assuming transition to occur abruptly at a specific point, normally chosen close to the centre of the transition region (see Rotta 1962). However, if the transition region occupies a high percentage of the surface, as in the case of flow over a turbine blade, the gradual change of the mean flow properties from the laminar to turbulent values is obviously of great importance for a reliable assessment of the boundary layer development to be made.

This is realised in the more recent prediction methods of McDonald & Fish (1973), Forrest (1977) and Abu-Ghannam & Shaw (1980). The former two methods are differential methods which solve the basic partial differential equations and ultimately results in predicted mean velocity profiles. These are then integrated numerically to give the boundary layer parameters. The method of Abu-Ghannam & Shaw is wholly empirical and is dependent on correlations of the mean flow parameters through transition.

The present method is different from the above techniques in that established integral methods for the laminar and turbulent boundary layers are used in conjunction with an intermittency modelled transition region. The advantage of this technique lies in the computational simplicity of integral prediction techniques and in the fact that the number of empirical correlations have been reduced to a minimum.
The two integral methods selected were those of Tani (1954), for the laminar boundary layer and Alber (1968), for the turbulent boundary layer. Alber's method was selected as it was reported to be one of the best methods presented at the Stanford Conference, Kline et al (1968) and Tani's method was selected as it was compatible with Alber's method, both being dissipation integral techniques. Both these methods are described in detail in Appendix 4.

In actual fact the transition model presented in this chapter is not dependent on the methods used for predicting the laminar and turbulent boundary layer components, provided that the methods predict the relevant flow parameters adequately. A comparison of the transition model using the methods of Thwaites (1949), for predicting the laminar boundary layer and Green et al (1977), for the turbulent boundary layer, is made between that of the Tani/Alber combination and is shown in fig. [7.4.8].

A complete boundary layer prediction scheme along with a fairly extensive graphics package was programmed to run on the same BBC micro-computer that was used for the data acquisition and control. The prediction scheme computes the development of the boundary layer from the leading edge of a plate through transition to the turbulent state. The input data was kept to a minimum and for arbitrary calculations the correlations of Abu-Ghannam & Shaw and that of the present author, equation 6.14 were used to define the onset of transition and the extent of the transition region respectively.
The transition model described below was then used to predict the development of the flow parameters through transition. A comparison of this model with a selection of the present data and that of Schubauer & Klebanoff (1956), Dhawan & Narasimha (1957) and Abu-Ghannam & Shaw is shown in figs. [7.4.1 - 7.4.8]

7.2 Transition model

As described in the previous chapter the flow within a transition region alternates between the laminar and turbulent flow states; the fraction of time spent in turbulent motion being governed by the intermittency function. The present intermittency function, which has been shown to be applicable in both zero and moderate non-zero pressure gradients, is defined by equation 6.4 in terms of the co-ordinates $\gamma$ versus $\zeta$.

Using this function the transition region is defined within the range $-2.3 < \zeta < 2.3$ corresponding to $0.01 < \bar{\gamma} < 0.99$. Therefore, provided the start and length of the transition region are defined, equations 6.4, 6.5 and 6.6 can be used to compute $\bar{\gamma}$ at any arbitrary position within the transition region.

Following Dhawan & Narasimha (1957) the mean transitional velocity profiles are represented by an intermittency weighted average of the separate laminar and turbulent components:

$$\left( \frac{u}{u_{\gamma}} \right)_{t} = (1 - \bar{\gamma}) \left( \frac{u}{u_{\gamma}} \right)_{L} + \bar{\gamma} \left( \frac{u}{u_{\gamma}} \right)_{T} \quad \ldots \ldots \quad 7.1$$

The boundary layer parameters through transition are defined as

$$\delta_{t}^{*} = \int_{0}^{\delta_{t}} \left\{ 1 - \left( \frac{u}{u_{\gamma}} \right) \right\} \, dy \quad \ldots \ldots \quad 7.2$$
where $\delta_t$ is the transitional boundary layer thickness and is taken as $\delta_L$ or $\delta_T$ whichever is the greatest.

Evaluation of these integrals using equation 7.1 results in

$$\delta_t = (1 - \gamma) \delta^*_L + \gamma(\delta^*_T) \quad \ldots \quad 7.5$$

$$\Theta_t = (1 - \gamma) \left\{ (1 - \gamma) \delta^*_L - \gamma \delta^*_L \right\} + \gamma(\gamma \delta^*_T - (1 - \gamma) \delta^*_T) + 2\gamma(1 - \gamma) \Phi(\delta_t) \quad \ldots \quad 7.6$$

where $\Phi(\delta_t) = \int_0^{\delta_t} \left\{ 1 - \left( \frac{u}{U_{\infty}} \right)_L \left( \frac{u}{U_{\infty}} \right)_T \right\} dy$.

Further, the skin friction coefficient through transition can be represented by:

$$C_{f_t} = (1 - \gamma) C_{f_L} + \gamma C_{f_T} \quad \ldots \quad 7.7$$

An additional boundary layer parameter predicted by this model is the energy thickness $\delta^{**}$, which is defined as

$$\delta^{**}_t = \int_0^{\delta_t} \left\{ \frac{u}{U_{\infty}} \right\}_L \left[ 1 - \left( \frac{u}{U_{\infty}} \right)_T^2 \right] dy \quad \ldots \quad 7.8$$

evaluation of this integral results in

$$\delta^{**}_t = (1 - \gamma) \left\{ (1 - \gamma)^2 \delta^{**}_L + \gamma(\gamma - 2) \delta^*_L \right\} + \gamma(\gamma^2 \delta^{**}_T + (\gamma^2 - 1) \delta^*_T) \quad 3\gamma(1 - \gamma) \Psi(\delta_t) \quad \ldots \quad 7.9$$

where $\Psi(\delta_t) = \int_0^{\delta_t} \left\{ 1 - \left( \frac{u}{U_{\infty}} \right)_L \left( \frac{u}{U_{\infty}} \right)_T \right\} dy$.

The derivation of this equation is given briefly in Appendix 3.

The energy thickness shape factor is then given by

$$H_{32_t} = \frac{\delta^{**}_t}{\Theta_t} \quad \ldots \quad 7.10$$
Evaluation of the mixed integral terms $F(\delta_t)$ and $Q(\delta_t)$ in equations 7.6 and 7.9 requires the laminar and turbulent mean velocity profiles to be fully specified.

**mean laminar velocity profile**

The Pohlhausen fourth-order polynomial velocity profile is assumed for the laminar mean velocity profile.

$$\left( \frac{u}{U_\infty} \right)_L = 2\left( \frac{\nu}{\delta_L} \right)^2 + 2\left( \frac{\nu}{\delta_L} \right)^3 + \frac{\lambda_P}{\theta} \left( \frac{\nu}{\delta_L} \right)^2 - 3\left( \frac{\nu}{\delta_L} \right)^2 + 3\left( \frac{\nu}{\delta_L} \right)^3 - \left( \frac{\nu}{\delta_L} \right)^\nu$$  \hspace{1cm} 7.11

where $\lambda_P = \frac{\delta_L^2}{\nu} \frac{dU_\infty}{dX}$  \hspace{1cm} 7.12

Tani's method for predicting the laminar boundary layer does not output the boundary layer thickness $\delta$. However the term $\lambda_\theta$ ie

$$\lambda_\theta = \frac{\delta_L^2}{\nu} \frac{dU_\infty}{dX} \hspace{1cm} 7.13$$

is available and is related to $\lambda_P$ through the Pohlhausen relationship.

$$\lambda_\theta = \frac{\lambda_P}{315} - \frac{\lambda_P^2}{945} - \frac{\lambda_P^3}{9072} \hspace{1cm} 7.14$$

The curve fits of Fraser (1979) are used to recast the subject of the above equation ie

$$\lambda_P = 10 \lambda_\theta \{ 6600 \lambda_\theta - 543 \} + (7 + 31 \lambda_\theta) \hspace{1cm} 7.15$$

For $\lambda_\theta > 0$

$$\lambda_P = \lambda_\theta \{ 73 + 109 \lambda_\theta + 790 \lambda_\theta^2 \} \hspace{1cm} 7.16$$

For $\lambda_\theta < 0$

The boundary layer thickness $\delta_L$ can then be obtained from equation 7.14.
mean turbulent velocity profile

The turbulent mean velocity profile is represented as a power law:

\[
\left( \frac{u}{U_\infty} \right)_T = \left( \frac{y}{\delta_T} \right)^{1/n}
\]

The exponent \( n \) being related to the turbulent velocity profile shape factor by

\[
n = \frac{2}{H_T - 1}
\]

and the thickness \( \delta_T \) represented by

\[
\delta_T = \frac{6T H_T (H_T + 1)}{H_T - 1}
\]

7.3 The computational model

The computational model was written in BBC BASIC and requires a minimum of input data to predict the development of a boundary layer from the leading edge of a body. The following input data is required:

(a) The freestream velocity distribution;
(b) The freestream turbulence level, \( T_u \);
(c) The plate length, \( L \);
(d) The reference velocity, \( U_0 \);
(e) The ambient pressure and temperature.

For computational convenience the freestream velocity distribution is input in the form

\[
\frac{U_\infty}{U_0} = E \left( \frac{X}{L} \right)^P + A + B \left( \frac{X}{L} \right) + C \left( \frac{X}{L} \right)^2 + D \left( \frac{X}{L} \right)^3
\]
which covers a convenient range of test flow conditions.

The method of Tani (1954) is then used to calculate the laminar boundary layer parameters from the leading edge through to the end of the transition region. Hence the values of $C_f_L$, $\delta_{L}$, $\theta_L$ and $\delta_{**L}$ are available throughout the transition region. The turbulent boundary layer calculation commences from the point at which transition starts through to the end of the plate. Therefore, the turbulent boundary layer parameters $C_f_T$, $\delta_{**T}$, $\theta_T$ and $\delta_{**t}$ are also available throughout the transition region. These values along with the numerical integration of the terms $F(\delta_T)$ and $Q(\delta_T)$ are then used to obtain the transition boundary layer parameters $C_f_T$, $\delta_{**T}$, $\theta_T$ and $\delta_{**t}$ from equations 7.7, 7.5, 7.6 and 7.9 respectively.

To start the turbulent calculation initial values of $\delta_{**T}$, $\Pi$ and $C_f_T$ are required. Whites skin friction correlation (equation 4.14), which requires the input of $R_\theta$ and $H_T$, is used to estimate the initial skin friction coefficient. Due to numerical difficulties the calculation could not be started at the flow origin i.e $\theta_T = 0$ therefore, the assumption that $\theta_T = \theta_{LS}/3$ is made for the starting value of $\theta_T$ at the point of transition. (Surprisingly the calculation procedure is relatively insensitive to the starting value of $\theta_T$). Following the work of Wygnanski et al (1976), who measured velocity profiles within turbulent spots, the initial shape factor is set to $H = 1.5$. With initial values of $C_f_T$, $\theta_T$ and $H_T$ defined, $\delta_{**T}$ and $\Pi$ are calculated from
Unlike the laminar boundary layer, the turbulent boundary layer mean flow parameters are affected by the magnitude of the turbulence level in the freestream. The empirical correlations of Bradshaw (1974) are incorporated into the turbulent calculation procedure to account for this effect i.e.

\[
\delta^*_T = \theta_T H_T \\
\Pi = 0.8 (\beta_T + 0.5) \\
\text{where } \beta_T = \frac{2 \delta_T^*}{Cf_T U_\infty} \frac{dU_\infty}{dx}
\]

No empirical relationship exists for the effect of freestream turbulence on the energy thickness. This should be realised when comparing the predicted values of \( \delta^* \) with the experimental data. However, for the range of freestream turbulence levels investigated the effect of the freestream turbulence is likely to be small as can be seen from the above equations.

The empirical correlation of Abu-Ghannam & Shaw (1980) which gives the momentum thickness Reynolds number at the start of transition as a function of the local pressure gradient parameter, \( \lambda_8 \), and the freestream turbulence level, \( Tu \), is included in the computational model to define the position at which transition commences. This correlation is given as:
\[ R_{Re} = 163 + \exp \left\{ f(\lambda_\theta) - f(\lambda_\theta) \frac{Tu}{6.91} \right\} \quad \ldots \ldots \quad 7.22 \]

where

\[ f(\lambda_\theta) = 6.91 + 12.75 \lambda_\theta + 63.64 (\lambda_\theta)^2 \quad \text{for} \quad \lambda_\theta < 0 \]

and

\[ f(\lambda_\theta) = 6.91 + 2.48 \lambda_\theta - 12.27 (\lambda_\theta)^2 \quad \text{for} \quad \lambda_\theta > 0 \]

The correlation due to the present author, equation 6.14 is then used to define the transition length. This equation is repeated below for completeness of this section.

\[
R_\theta = \left[ 270 - \frac{250 Tu^{3.5}}{1 + Tu^{3.5}} \right] \left[ \frac{1}{1 + 1710 (-\lambda_\theta)^{1.4} \exp \sqrt{1 + Tu^{3.5}}} \right] \times 10^3
\]

At present the model is restricted to zero and adverse pressure gradient flows as the transition length correlation is only applicable to such flows.

The computation model also incorporates a fairly extensive graphics package which can be used to compare the quality of the prediction to experimental data or merely to observe the prediction of the boundary layer parameters in the case of an arbitrary calculation.

The graphics software was written to enable a hard copy of the screen graphics to be obtained from a graph plotter which is controlled through the BBC microcomputer user port. Such printouts are shown in figures 7.4.1 - 7.4.8.
7.4 Model Performance

The validity of the model presented in this chapter was tested against a sample of the present data and the data of Dhawan & Narasimha (1957), Schubauer and Klebanoff (1956) and Abu-Ghannam & Shaw (1980). To make a fair assessment of the transition length correlation (equation 6.14), the experimentally measured position of the transition onset was read in as part of the input data.

Figures 7.4.1 (a) and 7.4.5 (b) show the predicted and measured boundary layer integral parameters and the mean velocity profiles for a representative sample of the present flows and also for the zero pressure gradient flow of Schubauer and Klebanoff. As can be observed from these figures the computational model predicts the boundary layer integral parameters and the velocity profiles very well for both the zero and adverse pressure gradient cases presented in the figures. Although the boundary layer velocity profiles and integral parameters have been predicted exceptionally well for the flow of Schubauer and Klebanoff the skin friction coefficient prediction is slightly lower than the experimentally measured values. However, due to difficulty in measuring the skin friction in a transitional flow, not too much emphasis should be placed on this observation.

Figures 7.4.6 and 7.4.7 show the present prediction against the experimentally measured data of Abu-Ghannam & Shaw for both a zero and an adverse pressure gradient flow. The zero pressure gradient flow is well predicted by the model, with the exception of the skin friction coefficient which again is slightly low, although not markedly so. The distribution of the skin friction
through the transition region for the adverse pressure gradient flow of the Abu-Ghannam & Shaw has been poorly represented by the model.

The reason for this lies, not in the transition model itself, but in the fact that the transition length has been predicted approximately 30% less than the experimentally measured value.

As mentioned in section 7.1 the integral methods used to compute the component laminar and turbulent parameters for the transition model are not important provided they are reliable and well established methods. Fig. [7.4.8] shows the transition model described in section 7.2, against the data of Dhawan & Narasimha, using two different combinations of integral methods for the computation of the component laminar and turbulent boundary layer parameters. The two combinations are

(i) Tani/Alber used for the previous predictions.

(ii) Thwaites/Green et al - Thwaites (1949) method being used for the laminar boundary layer computation and Green et al (1977) lag entrainment method being used for the turbulent boundary layer computation.

As can be seen from this figure the transition model performs equally well irrespective of the combination chosen.

In general the transition model predict the flows very well but the method is crucially dependent on the accurate prediction of the onset and extent of the transition region. This is not peculiar to this particular method but would be as important for any method which uses transition onset and length correlations as a basis.
Flow 3 details in table 6.2

---

Fig. 7.4.1 (a) Boundary layer prediction of present Flow 3

Fig. 7.4.1 (b) Predicted mean velocity profiles through transition
Flow 4 details in table 7.2

Fig. 7.4.2 (a) Boundary layer prediction of present Flow 4

Fig. 7.4.2 (b) Predicted mean velocity profiles through transition
Flow 6 Details in table 6.2

Fig. 7.4.3 (a) Boundary layer prediction of present Flow 6

Fig. 7.4.3 (b) Predicted mean velocity profiles through transition
Flow 14 Details in table 6.2

Fig. 7.4.4 (a) Boundary layer prediction of present Flow 14

Fig. 7.4.4 (b) Predicted mean velocity profiles through transition
Fig. 7.4.5 (a) Boundary layer prediction of Schubauer & Klebanoff (1956 zero pressure gradient flow)

Tu = 0.03

x_s = 1600 mm

du

dx = 0

U_o = 24.4 m/s

Fig. 7.4.5 (b) Predicted mean velocity profiles through transition
$Tu = 1.2\%$
$x_S = 320\ mm$
\[
\frac{dU}{dx} = 0
\]
$U_0 = 22\ m/s$

Fig. 7.4.6 Boundary layer prediction of Abu-Ghannam & Shaw (1980)

zero pressure gradient flow
$T_u = 1.2\%$
$x_s = 310 \text{ mm}$
$\frac{dU}{dx} = 2.30 \frac{1}{s}$
$U_o = 20.63 \text{ m/s}$

Fig. 7.4.7 Boundary layer prediction of Abu-Ghannam & Shaw (1980)
Adverse pressure gradient flow
$Tu = 1.3$
$x_S = 460 \text{ mm}$
$\frac{dU}{dx} = 0$
$U_0 = 13.1 \text{ m/s}$

Fig. 7.4.8 Boundary layer prediction of Dhawan & Narasimha (1958)
zero pressure gradient flow
CONCLUSIONS

1. The data acquisition and control system, based on the BBC microcomputer with BEEBEX Eurocard extension, was found to be completely satisfactory for the measurement of the mean flow variables in the laminar, turbulent and transitional boundary layers. The addition of this system, with analogue input and control, greatly enhanced the rate at which reliable data could be gathered and analysed. A further benefit lies in the ability to store the prime data in an organised manner for subsequent manipulation and graphical display. The data acquisition, control and data reduction software package developed, being interactively instructive, is extremely simple to use.

2. The flow over the test plate in the boundary layer wind tunnel facility has been improved to enable "free transition" values of $R_{x_S}$ which concur with those of previous researchers.

3. A series of flows with different combinations of pressure gradient and freestream turbulence level were successfully set up and the boundary layer development for each case was recorded. Measurements were restricted to the plate centre line to remove the possibility of tunnel side wall effects influencing the results.

4. The system developed for measuring the intermittency function $\gamma$ was very successful in giving reliable and repeatable measurements of the intermittency distribution $\bar{\gamma}(x)$ through the transition region.
5. The general boundary layer is well qualified by the near wall intermittency value. At intermittencies below $\bar{\gamma} = 0.01$ the mean flow is characteristically laminar and at intermittencies above $\bar{\gamma} = 0.99$ the mean flow is characteristically turbulent. At $\bar{\gamma}$ values between 0.01 and 0.99 the boundary layer is transitional and can be represented by an intermittency weighted function of the laminar and turbulent velocity components ie equation 7.1. The alternate switching process between the laminar and turbulent flow states within a transition region has been corroborated in a qualitative manner by observations of the instantaneous velocity signal within a transitional boundary layer, see fig. [6.4.1].

6. On the strength of the present data, the concept of statistical similarity of transition regions is shown to remain intact for moderate non-zero pressure gradients.

7. The mean intermittency distribution through a transition region is well represented by the normal distribution function and the statistical similarity is best illustrated when the intermittency $\bar{\gamma}$ is plotted against the normalised streamwise co-ordinate $\zeta$.

8. With the transition region defined within the limits $0.01 < \bar{\gamma} < 0.99$ then the standard derivation $\sigma$ of the intermittency data on the $\gamma (\zeta)$ distribution can be related directly to the transition length by equation 6.6 (see also fig. [6.6.2]).
9. The present results have shown that an adverse pressure gradient will promote early transition whilst a favourable pressure gradient will delay transition. This concurs with results from stability theory, Schlichting (1979), which indicate that adverse pressure gradients have a destabilising effect on the flow and that favourable pressure gradients have a stabilising effect.

10. For the present range of freestream turbulence levels tested, increasing the freestream turbulence has the effect of advancing the onset of transition. However, this effect becomes less significant with increasing adverse pressure gradient.

11. The length over which the breakdown process from laminar to turbulent flow occurs, ie the transition length, is greatly affected by both the freestream pressure distribution and the freestream turbulence level. In all cases the transition region is shortened by the influence of an adverse pressure gradient. From the present favourable pressure gradient/transition length data available, it would appear that the transition length is increased by the influence of a favourable pressure gradient, however this observation is speculative as it is based on only one flow condition. In adverse pressure gradients, it has been shown that increasing the freestream turbulence, up to $Tu = 1\%$, has the effect of increasing the transition length. In this range, ie $0 < Tu < 1$, the freestream turbulence level has a greater effect in advancing the start of transition than the end of transition hence the overall transition length is increased. Increasing $Tu$ above $1\%$ reverses this trend, in consequence of the start of transition
approaching its asymptotic minimum position while the end of transition is still being significantly advanced.

12. A correlation which accounts for the combined effect of freestream turbulence and adverse pressure gradient on the transition length Reynolds number $R_0$ is presented. (Equation 6.14)

13. A general boundary layer integral prediction scheme for two-dimensional incompressible flows, which incorporates the new transition length correlation, has been developed. This prediction scheme uses existing integral techniques for the laminar and turbulent boundary layers in conjunction with an intermittency modelled transition region. The computational model was programmed to run on a BBC microcomputer and was tested against a representative sample of the present data and a number of flat plate test cases and is shown to model the development of the transitional boundary layer exceptionally well.
SUGGESTIONS FOR FUTURE WORK

A limitation of the present boundary layer prediction model is its inability to compute the development of a transitional boundary layer in a case where laminar separation is predicted within the transition region. This stems from the fact that accurate values of the laminar boundary layer parameters are required, in the computation procedure, through to the end of transition.

It has recently been brought to the author's attention that this limitation may present a problem when applying the present model to a flow with a velocity distribution typical of that which exists on the suction surface of a gas turbine blade. An interesting development of the present work would therefore be to set up conditions to simulate the flow over a turbine blade and to make detailed measurements of the boundary layer development on the test surface. [The present boundary layer wind tunnel, with perhaps a few minor modifications, would be capable of reproducing the type of velocity distribution required to simulate such a flow].

Two questions which immediately spring to mind, which may be elucidated by such an investigation:

(i) Does this predicted laminar separation actually occur within the transition region?
(ii) If it does occur, how can the effect be modelled and accounted for in a computational procedure?
To answer these questions, condition sampling techniques would have to be employed to measure the separate laminar and turbulent velocity profiles in and out of turbulent patches within the transition region. This would necessitate the use of a fast data acquisition system capable of storing large amounts of data. Although the sample rate of the current BBC micro based system could be improved through the use of machine code routines, the limited amount of memory available would strictly limit its usefulness for such an investigation and therefore the present instrumentation would have to be enhanced by the addition of a more powerful microcomputer, possibly for example, the IBM PC with 512 K of available RAM.

Such an experimental program could also be extended to provide further data on the effect of local parameters on the location and extent of the transition region.
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APPENDIX 1

Estimation of the experimental uncertainty in the measured boundary layer integral thicknesses
APPENDIX 1 Experimental uncertainty in integral thicknesses

A1.1 Error due to uncertainty in velocity measurement

As described in section 4.4, when linearising the signal from a hot wire probe using the DISA 55M25 lineariser there is an inherent parabolic error, usually with the maximum close to the centre of the linearised region and tailing off to zero at the maximum and minimum velocities, as shown in figure 4.4.1. The analysis which follows describes the effect such an error would induce on the boundary layer integral parameters in both a laminar (parabolic type velocity profile) and a turbulent \((1/n)\) th. power type velocity profile) boundary layer.

The error is assumed to be zero at both \(\bar{u}/U_\infty = 0\) and \(\bar{u}/U_\infty = 1\) with a maximum at \(\bar{u}/U_\infty = 0.5\) as shown below:

\[
\Sigma = A + B \left(\frac{\bar{u}}{U_\infty}\right) + C \left(\frac{\bar{u}}{U_\infty}\right)^2
\]

Writing \(U = \left(\frac{\bar{u}}{U_\infty}\right)\) and applying the boundary conditions

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(i) \( U = 0, \Sigma = 0 \)

(ii) \( U = 1, \Sigma = 0 \)

(iii) \( U = 0.5, \frac{d\Sigma}{dU} = 0, \Sigma = \Sigma_m \)

results in the local error being represented by:

\[ \Sigma = 4 \Sigma_m (U - U^2) \]

If this error is now introduced into the local velocity measurements ie.

\[ U' = U + \Sigma U \]

then the error introduced into the displacement, momentum and energy thicknesses are respectively

\[ S^* = \int_0^1 (1 - U') d\gamma \quad \Theta' = \int_0^1 U'(1 - U') d\gamma \]

and

\[ S^{**} = \int_0^1 U'(1 - U'^2) d\gamma \]

Considering first the displacement thickness, \( S^* \)

\[ S^* = \int_0^1 (1 - U - \Sigma U) d\gamma \]

\[ = \int_0^1 [1 - U - 4 \Sigma_m (U - U^2)] d\gamma \]

\[ = \int_0^1 (1 - U) d\gamma - 4 \Sigma_m \int_0^1 (U^2 - U^3) d\gamma \]

\[ \therefore \Delta S^* = S^* - S^* = -4 \Sigma_m \int_0^1 U^2 - U^3 d\gamma \]
Hence:

\[
\frac{\Delta \xi^*}{\xi^*} = \frac{-4 \Sigma m \int_0^1 (U^2 - U^3) \, d\gamma}{\int_0^1 (1 - U) \, d\gamma}, \tag{A1.1}
\]

Considering now the momentum thickness, \( \Theta \)

\[
\Theta' = \int_0^1 \left[ (U + \Sigma U) \left[ 1 - (U + \Sigma U) \right] \right] \, d\gamma
\]

\[
= \int_0^1 \left[ (U + 4 \Sigma m (U-U^3)) \left[ 1 - (U + 4 \Sigma m (U-U^3)) \right] \right] \, d\gamma
\]

\[
= \int_0^1 \left[ (U + 4 \Sigma m U^2 - 4 \Sigma m U^3) \left[ 1 - U - 4 \Sigma m U^2 + 4 \Sigma m U^3 \right] \right] \, d\gamma
\]

Neglecting power terms in \( \Sigma m \) results in:

\[
\Theta' = \int_0^1 \left[ (U - U^2) + 4 \Sigma m U^2 - 12 \Sigma m U^3 + 8 \Sigma m U^4 \right] \, d\gamma
\]

\[
\Rightarrow \Theta' = \Theta + 4 \Sigma m \int_0^1 U^2 (1 - 3U + 2U^2) \, d\gamma
\]

\[
\therefore \Delta \Theta = \Theta' - \Theta = 4 \Sigma m \int_0^1 U^2 (1 - 3U + 2U^2) \, d\gamma
\]

Hence:

\[
\frac{\Delta \Theta}{\Theta} = \frac{4 \Sigma m \int_0^1 U^2 (1 - 3U + 2U^2) \, d\gamma}{\int_0^1 U (1 - U) \, d\gamma}, \tag{A1.2}
\]

Similarly if power terms in \( \Sigma m \) are neglected then:

\[
\frac{\Delta \xi^{**}}{\xi^{**}} = \frac{4 \Sigma m \int_0^1 U^2 (1 - U + 3U^2 - 3U^3) \, d\gamma}{\int_0^1 U (1 - U^2) \, d\gamma}, \tag{A1.3}
\]
Assuming a parabolic "laminar" velocity profile ie.

\[ U = 2 \gamma - \gamma^2 \]

Substituting this into equations Al.1, Al.2, and Al.3 and evaluating, results in:

\[
\frac{\Delta S^*}{S} = -0.92 \Sigma_m
\]
\[
\frac{\Delta \theta}{\theta} = -0.78 \Sigma_m
\]
\[
\frac{\Delta S^{**}}{S} = -0.70 \Sigma_m
\]

Substituting a \( \frac{1}{7} \) th. power "turbulent" velocity profile into equations Al.1, Al.2 and Al.3 and evaluating, results in:

\[
\frac{\Delta S^*}{S} = -2.5 \Sigma_m
\]
\[
\frac{\Delta \theta}{\theta} = -2.2 \Sigma_m
\]
\[
\frac{\Delta S^{**}}{S} = -1.9 \Sigma_m
\]

Therefore if the maximum error ie. \( \Sigma_m \) in the velocity measurements is 1\% then the corresponding errors in \( S^*, \theta, \Sigma \) for a parabolic velocity profile would be -0.92\%, 0.78\% and -0.70\% respectively and -2.5\%, -2.2\% and -1.9\% respectively for a \( \frac{1}{7} \) th. power velocity profile. The negative sign shows that if the error \( \Sigma_m \) is positive then the integral thicknesses are underestimated. Figure Al.1.1 shows a parabolic profile with and without a 5\% maximum error included (ie. \( \Sigma_m = 0.05 \)) and figure Al.1.2 shows the corresponding profiles of \( (1-U) \), \( U(1-U) \) and \( U(1-U^2) \).
A1.2 Error due to uncertainty in y datum

A numerical error analysis assuming $y'$ was underestimated by 1% showed that the corresponding error in all the boundary layer integral thicknesses was approximately -1%. The y datum can be set, at the very worst, to an accuracy of ±0.1 mm therefore, if the boundary layer thickness is small this may constitute a non-negligible error in the integral parameters (in a 10mm thick boundary layer the error in the integral thicknesses would, at worst, be 1% due to the uncertainty in setting the y datum). This error obviously diminishes as the boundary layer thickens.
Figure Al.1.1 Velocity profile

Figure Al.1.2
APPENDIX 2

Description of a microcomputer based system for setting up the DISA 55M25 lineariser
APPENDIX 2 Microcomputer based system for setting up the DISA 55M25 Lineariser

The procedure for linearising the signal from a hot wire probe using the DISA 55M25 lineariser is fairly complex and usually very time consuming. After the basic functions have been set, as described in the user manual, the signal from the probe is linearised via the 55M25 by a trial and error iterative process. The probe is first exposed to a known velocity at the high end of the calibration range and then the 'Gain High' control on the 55M25 is adjusted to give the required output on a digital voltmeter, DVM, (usually the voltage output from the lineariser is made to correspond to a convenient fraction of the flow velocity, in this case \(\frac{1}{10}\) th.). The probe is then exposed to a known velocity at the low end of the calibration range and if the reading on the DVM does not correspond to the relevant velocity the 'Exponent Factor' control, on the 55M25, is adjusted. The probe is then again exposed to the high velocity and the DVM reading is checked. This process continues until a satisfactory linearisation is achieved.

The problem with this procedure is that, although adjustment of the various controls on the 55M25 have varying degrees of influence on different regions within the calibration range, eg. the Gain High control affects the high velocity end of the range most significantly, each adjustment does in fact have some influence over the entire linearisation range. However the effect of such adjustments can only be assessed at one point at any one time.

Therefore, for example when examining a reading on the DVM at the high end of the calibration range and making an adjustment to the Gain High control, there is no indication given of the effect this adjustment has on points at other positions within the linearisation
region.

To overcome this problem and to speed up the linearisation process a microcomputer based system was developed which enabled a range of calibration points to be continuously fed into the lineariser input. The corresponding values output from the lineariser are then displayed on a monitor along with a line which corresponds to the 'ideal' linearised signal. The set up for this system is shown in figure A2.1 and a print out of the software required to run the system along with a flow diagram is given in figures A2.2 & A2.3. (The BUFFER between the DAC and the Signal Conditioner, shown in figure A2.1, was required to overcome impedance matching problems encountered).

Briefly, a set of calibration points are obtained from a single run of the tunnel, in the relevant velocity range (i.e. raw hot wire voltages and corresponding pitotstatic readings). These values are then manually fed into the BBC microcomputer. After the points have been suitably conditioned, within the software, they are output to the lineariser via a Digital-to-Analogue Converter, DAC, and then retrieved from the lineariser via an Analogue-to-Digital Converter, ADC, in a closed loop. These points are continuously displayed on a graph of $V_{in}$ (voltage into lineariser) against $V_{out}$ (voltage out of lineariser), along with a line which corresponds to the 'ideal' linearised signal. The effect on each point of a single adjustment to the lineariser controls can then be viewed, almost immediately, and an optimum setting can fairly easily be obtained.
Fig. A.2.1 Schematic diagram of apparatus used for setting up the DISA 55M25 lineariser
10 CLOSE#0
20 MODE7
30 CLS
40 DIM P(11), Vbr(11), Vdac(11), Vadc(11)
50 DIM Voutpl(11), Vinp1(11)
60 REM FROM LINE PROG
70 PRINTTAB(4,8)CHR$132"INPUT AIR TEMP IN DegC"
80 INPUTTAB(32,8) T
90 PRINTTAB(4,12)CHR$132"INPUT ATMOS. PRESS IN mmHg"
100 INPUTTAB(32,12) Z
110 Rho=8.46556*2/(T+273)
120 CLS
130 PRINTTAB(0.2)CHR$133"INPUT CORRESPONDING H.W. VOLTAGES"
140 PRINTTAB(8,4)CHR$133"HWP0SITIVE THE DYNAMIC PRESSURES GIVEN"
150 PRINT: PRINT: PRINTCHR$134" Dynamic press. H.W. Bridge"
160 PRINTCHR$134" mmHg Voltage"
160 I=0
170 FOR I = 1 TO 10
180 PRINTTAB(8,10+X): P(I)
190 INPUTTAB(24,10+X) Vbr(I)
200 X=X+1
210 NEXT
220 MODE12
230 VDU 19,7,3,0,0,0
240 VDU 19,2,4,0,0,0
250 MOVE125,825:DRAW125,125:DRAW1225,125
260 VDU4
270 MOVE125,92:PRINT"V"
280 MOVE125,77:PRINT"H"
290 MOVE125,62:PRINT"V"
300 MOVE125,47:PRINT"R"
310 MOVE125,32:PRINT"S"
320 MOVE125,17:PRINT"T"
330 MOVE125,0:PRINT"U"
340 MOVE125,-12:PRINT"V"
350 MOVE125,-27:PRINT"W"
360 MOVE125,-42:PRINT"X"
370 MOVE125,-57:PRINT"Y"
380 MOVE125,-72:PRINT"Z"
390 MOVE125,-87:PRINT"A"
400 MOVE125,-102:PRINT"B"
410 MOVE125,-117:PRINT"C"
420 MOVE125,-132:PRINT"D"
430 MOVES50,750:PRINT"V"
440 MOVES50,700:PRINT"V"
450 MOVES50,650:PRINT"V"
460 MOVES50,600:PRINT"V"
470 MOVES50,550:PRINT"V"
480 MOVES50,500:PRINT"V"
490 MOVES50,450:PRINT"V"
500 MOVES50,400:PRINT"V"
510 MOVES50,350:PRINT"V"
520 MOVES50,300:PRINT"V"
530 MOVES50,250:PRINT"V"
540 MOVES50,200:PRINT"V"
550 MOVES50,150:PRINT"V"
560 MOVES50,100:PRINT"V"
570 MOVES50,50:PRINT"V"
580 MOVES50,0:PRINT"V"
590 MOVE125,98:PRINT"V"
600 MOVE125,93:PRINT"V"
610 MOVE125,88:PRINT"V"
620 MOVE125,83:PRINT"V"
630 MOVE125,78:PRINT"V"
640 MOVE125,73:PRINT"V"
650 MOVE125,68:PRINT"V"
660 MOVE125,63:PRINT"V"
670 MOVE125,58:PRINT"V"
680 MOVE125,53:PRINT"V"
690 MOVE125,48:PRINT"V"
700 MOVE125,43:PRINT"V"
710 MOVE125,38:PRINT"V"
720 MOVE125,33:PRINT"V"
730 MOVE125,28:PRINT"V"
740 MOVE125,23:PRINT"V"
750 MOVE125,18:PRINT"V"
760 MOVE125,13:PRINT"V"
770 MOVE125,8:PRINT"V"
780 MOVE125,3:PRINT"V"
790 MOVE125,0:PRINT"V"
800 Ydatum1=((Vbr(2)-2)*233.34)+125
810 Ydatum2=((Vbr(18)-2)*233.34)+125
820 Xdatum1=((SQR(2.91*P(2)/Rho))/10)*550)+125
830 Xdatum2=((SQR(2.91*P(10)/Rho))/10)*550)+125
840 MOVE125, 1 = Ydatum1, Ydatum2
850 10
860 AX=OPENUP "CU-DACE 8C000"
870 FOR I = 1 TO 10
880 Vdac(I)=Vbr(I)*50.5
890 PTR#A=Vdac(I)
900 Vsum0
910 PTR#A=0
920 FOR K = 1 TO 10
930 V2=BGET#A
940 Vsum+=Vsum+V2
950 NEXT
960 Vadc(I)=(Vsum/I)*2/255
970 Vinp1(I)=((Vbr(I)-2)*233.34)+125
980 EC0L 0.1
990 Voutpl(I)=((Vadc(I)+550)+125
100 PLOT 69, Voutpl(I), Vinp1(I)
101 NEXT
102 TIME=0: REPEAT UNTIL TIME=50
103 FOR I=1 TO 10
104 PLOT 71, Voutpl(I), Vinp1(I)
105 NEXT
106 GOT0478
107 DATA 0.0,5,1.0,2.25,4.0,6.25,9.0,12.25,16.0,20.25
108 DATA 0,1,2,3,4,5,6,7,8,9,10

Figure A2.2 Print out of software for DISA 55M25 set up system
START

READ IN CALIBRATION POINTS
  ie. hot wire voltages &
  corresponding pitot readings

DISPLAY CALIBRATION GRAPH
  along with the ideal
  linearised line

CONVERT H.W. VOLTAGES TO
DIGITAL VALUES via SOFTWARE
CALIBRATION

FOR I = 1 to N

FOR EACH CALIBRATION H.W.
VOLTAGE, OUTPUT THE
CORRESPONDING DIGITAL VALUE
TO THE LINEARISER via THE
DAC

RETRIEVE'LINEARISED POINTS
FROM LINEARISER via THE
ADC

CONVERT DIGITAL VALUES TO
ANALOGUE EQUIVALENT WITHIN
SOFTWARE

SCALE AND PLOT ON
CALIBRATION GRAPH

LOOP COMPLETE

FOR I = 1 to N

ERASE CALIBRATION
POINT ON GRAPH

LOOP COMPLETE

Figure A2.3 Software flow diagram
APPENDIX 3

Derivation of equation 7.9 for the energy thickness in a transitional boundary layer
APPENDIX 5 Derivation of $S_t^*$

As described by Dhawan & Narasimha (1958) the transitional boundary layer mean velocity profile can be defined as:

$$
\left( \frac{\bar{u}}{U_\infty} \right)_t = (1 - \delta) \left( \frac{\bar{u}}{U_\infty} \right)_L + \delta \left( \frac{\bar{u}}{U_\infty} \right)_T
$$

Writing:

$$
\bar{u} = \left( \frac{\bar{u}}{U_\infty} \right)_t
$$

and defining:

$$
S_{t}^{**} = \int_{S_t} U \left[ 1 - (U)^2 \right] \, dy
$$

then:

$$
S_{t}^{**} = \int_{S_t} \left[(1 - \delta) U_L + \bar{\delta} U_T \right] \left[1 - \left((1 - \delta) U_L + \bar{\delta} U_T \right)^2\right] \, dy
$$

Considering terms in $U_L$ only:

$$
\int_{0}^{S_t} (U_L - U_L^3 + 3 \delta U_L^3 - 3 \delta^2 U_L^3 - \bar{\delta} U_L + \bar{\delta}^3 U_L^3) \, dy
$$

adding and subtracting $(1 - \delta)^2 U_L$ gives:

$$
(1 - \delta) \int_{0}^{S_t} \left[ (1 - \delta)^2 U_L - (1 - \delta)^2 U_L^3 + U_L - (1 - \delta)^2 U_L \right] \, dy
$$

$$
= (1 - \delta) \left\{ (1 - \delta)^2 S_L^{**} + \int_{0}^{S_t} [U_L - (1 - \delta)^2 U_L] \, dy \right\}
$$

after some algebra:

$$
= (1 - \delta) \left\{ (1 - \delta)^2 S_L^{**} + (1 - \delta)^2 S_L^* - S_L^* + \int_{0}^{S_t} [2 \bar{\delta} - \bar{\delta}^3] \, dy \right\}
$$

$$
= (1 - \delta) \left\{ (1 - \delta)^2 S_L^{**} + (1 - \delta)^2 S_L^* - S_L^* + \bar{\delta} (2 - \bar{\delta}) S_t \right\}
$$

--- A3.2
From A3.1 considering terms in $U_T$ only, ie.

$$\int_0^{S_t} (\gamma U_T - \gamma^3 U_T) \, dy$$

$$= \bar{\gamma} \int_0^{S_t} \left[ \bar{\gamma}^2 (U_T - U_T^3) + U_T - \bar{\gamma}^2 U_T \right] \, dy$$

$$= \bar{\gamma} \left\{ \bar{\gamma}^2 S_T^{**} + \int_0^{S_t} (U_T - \bar{\gamma}^2 U_T) \, dy \right\}$$

$$= \bar{\gamma} \left\{ \bar{\gamma}^2 S_T^{**} + \int_0^{S_t} \left[ \bar{\gamma}^2 (1-U_T) + U_T - \bar{\gamma} \right] \, dy \right\}$$

$$= \bar{\gamma} \left\{ \bar{\gamma}^2 S_T^{**} + \bar{\gamma}^2 S_T^{**} + \int_0^{S_t} \left[ U_T - \bar{\gamma}^2 \right] \, dy \right\}$$

$$= \bar{\gamma} \left\{ \bar{\gamma}^2 S_T^{**} + \bar{\gamma}^2 S_T^{**} - \bar{S}^* + (1-\bar{\gamma}^2) S_T \right\} \quad \text{A3.3}$$

From A3.1 considering terms in $U_L U_T$ only, ie.

$$\int_0^{S_t} \left[ -3 \bar{\gamma} U_L^2 U_T + 6 \bar{\gamma}^2 U_L^3 U_T - 3 \bar{\gamma}^2 U_L U_T^2 - 3 \bar{\gamma}^3 U_L^2 U_T + 3 \bar{\gamma}^3 U_L U_T^2 \right] \, dy$$

$$= -3 \bar{\gamma} (1-\bar{\gamma}) \int_0^{S_t} \left[ (1-\bar{\gamma}) U_L^2 U_T + \gamma U_L U_T^2 \right] \, dy$$

$$= -3 \bar{\gamma} (1-\bar{\gamma}) \int_0^{S_t} \left[ ((1-\bar{\gamma}) U_L + \bar{\gamma} U_T) U_L U_T \right] \, dy$$

$$= -3 \bar{\gamma} (1-\bar{\gamma}) \int_0^{S_t} \left[ U_L U_L U_T \right] \, dy \quad \text{where } U_T = (1-\bar{\gamma}) U_L + \gamma U_T$$

$$= -3 \bar{\gamma} (1-\bar{\gamma}) \int_0^{S_t} \left[ 1 - U_L U_L U_T \right] \, dy$$

$$= -3 \bar{\gamma} (1-\bar{\gamma}) S_t + 3 \gamma (1-\bar{\gamma}) \int_0^{S_t} \left[ 1 - U_L U_L U_T \right] \, dy \quad \text{A3.4}$$
Grouping A3.2, A3.3, A3.4 gives:

\[ S_t^{**} = (1-\delta) \left[ (1-\delta)^2 S_{L}^{*} + (1-\delta)^2 S_{L}^{*} - S_{L}^{*} + \delta(2-\delta) S_{t} \right] + \delta \left[ \delta^2 S_{T}^{**} + \delta^2 S_{T}^{*} - S_{T}^{*} + (1-\delta^2) S_{t} \right] - 3\delta (1-\delta) S_{t} + 3\delta (1-\delta) \int_{0}^{+\delta} \left[ 1-u_{LUU} \right] dy \]

Considering all terms in \( S_t \) ie.

\[ (1-\delta) \delta (2-\delta) S_{t} + \delta (1-\delta^2) S_{t} - 3\delta (1-\delta) S_{t} \]

\[ = (3\delta - 3\delta^2 - \delta^3 - 3\delta + 3\delta^2) S_{t} \]

\[ = 0 \]

Giving finally:

\[ S_t^{**} = (1-\delta) \left[ (1-\delta)^2 S_{L}^{*} + \delta(2-\delta) S_{L}^{*} \right] + \delta \left[ \delta^2 S_{T}^{**} + (\delta^2-1) S_{T}^{*} \right] + 3\delta (1-\delta) \int_{0}^{+\delta} \left[ 1-u_{LUU} \right] dy \]
APPENDIX 4
Integral prediction methods for laminar and turbulent boundary layers.
APPENDIX 4 Integral prediction methods for laminar and turbulent boundary layers

A4.1 Tani's(1954) method for laminar boundary layers

Tani's method makes use of both the momentum integral and energy integral equations in an approximate solution for the laminar boundary layer. Tani assumes that the velocity profiles belong to a one parameter family of curves but adopts a new profile parameter in favour of the usual Pohlhausen/Thwaites parameter, \( \lambda \). The relationship between the new profile parameter and \( \lambda \) is derived from the momentum integral and energy integral equations and used in the solution.

The basic equations:

The basic equations for a steady two dimensional, incompressible laminar boundary layer are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{A4.1}
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} = U_{\infty} \frac{d u}{d x} + \nu \frac{\partial^2 u}{\partial y^2} \quad \text{A4.2}
\]

A4.1 and A4.2 are the continuity and momentum equations respectively.

The momentum integral equation is obtained by integrating A4.2 from \( y = 0 \) to \( y = \delta \) and results in:

\[
\frac{U_{\infty}}{\nu} \frac{d \delta^2}{d x} + 2 \frac{\delta^2}{\nu} \frac{d U_{\infty}}{d x} \left( 2 + \frac{\delta^*}{\Theta} \right) = \frac{2 \Theta}{U_{\infty}} \left( \frac{\partial U_{\infty}}{\partial y} \right)_{y=0} \quad \text{A4.3}
\]

and the energy integral equation is obtained by multiplying A4.2 through by \( U_{\infty} \) and integrating w.r.t. \( y \) from \( y = 0 \) to \( y = \delta \) resulting in:

\[
\frac{U_{\infty}}{\nu} \frac{d \delta^{**2}}{d x} + 6 \frac{\delta^{**2}}{\nu} \frac{d U_{\infty}}{d x} = 4 \frac{\delta^{**2}}{U_{\infty}^2} \int_0^\delta \left( \frac{\partial U_{\infty}}{\partial y} \right)^2 dy \quad \text{A4.4}
\]
Following Pohlhausen, Tani assumed a laminar velocity profile in the form:

\[
\frac{u}{u_\infty} = a_0 + a \left( \frac{y}{y_0} \right) + b \left( \frac{y}{y_0} \right)^2 + c \left( \frac{y}{y_0} \right)^3 + d \left( \frac{y}{y_0} \right)^4 \quad \text{--- A4.5}
\]

However, in contrast to Pohlhausen the usual condition:

\[
y = 0 : \quad \frac{\partial^2 u}{\partial y^2} = - \frac{u_\infty}{\nu} \frac{du_\infty}{dx}
\]

which states that equation A4.2 is satisfied at the wall, is dropped so that the coefficient, \( a_0 \), remains undetermined. This coefficient is now adopted as the profile parameter and the velocity profiles are then represented by:

\[
\left( \frac{u}{u_\infty} \right) = \left( \frac{y}{y_0} \right)^2 \left[ 6 - 8 \left( \frac{y}{y_0} \right) + 3 \left( \frac{y}{y_0} \right)^2 \right] + a \left( \frac{y}{y_0} \right)^3 \left[ 1 - \frac{y}{y_0} \right]^{\frac{1}{2}} \quad \text{--- A4.6}
\]

Tani then introduced the non-dimensional quantities:

\[
\frac{\Theta^*}{\Theta} = D ; \quad \frac{\Theta}{\Theta} = E ; \quad \frac{\Theta^{**}}{\Theta} = F \quad \text{--- A4.7}
\]

\[
H_{12} = \frac{\Theta^*}{\Theta} ; \quad H_{32} = \frac{\Theta^{**}}{\Theta} \quad \text{--- A4.8}
\]

\[
\frac{2}{U_\infty} \left( \frac{\partial u}{\partial y} \right)_{y=0} = P ; \quad \frac{4}{U_\infty} \int_0^1 \left( \frac{\partial u}{\partial y} \right) dy = Q \quad \text{--- A4.9}
\]

Using A4.7, A4.8 and A4.9, equations A4.3 and A4.4 can be rewritten in the form:

\[
\frac{U_\infty}{\nu} \frac{d\Theta^2}{dx} + 2 \left( 2 + H_{12} \right) \frac{\Theta^2}{\nu} \frac{du_\infty}{dx} = P \quad \text{--- A4.10}
\]

\[
\frac{U_\infty}{\nu} \frac{dH_{32} \Theta^2}{dx} + 6 \frac{H_{32} \Theta^2}{\nu} \frac{du_\infty}{dx} = Q \quad \text{--- A4.11}
\]

where

\[
D = \frac{2}{5} - \frac{a}{20} \quad \text{--- A4.12}
\]

\[
E = \frac{4}{35} + \frac{a}{105} - \frac{a^2}{252} \quad \text{--- A4.13}
\]

\[
F = \frac{876}{5005} + \frac{73}{5005} a - \frac{23}{5460} a^2 - \frac{a^3}{2860} \quad \text{--- A4.14}
\]
\[ P = 2aE \quad \text{A4.15} \]
\[ Q = \left\{ \left( \frac{4}{3} \right) F \right\} (4.8 - 4a + 3a^2) \quad \text{A4.16} \]

Tani also makes use of the approximation
\[ \frac{\Theta^2}{V_0} = \frac{0.44}{U_\infty^4} \int_0^x U_\infty^5 \, dx \quad \text{A4.17} \]

which is almost identical to Thwaites (1949) quadrature. However, Tani derived this form directly from the energy integral equation on the assumption that the variations in \( H_{32} \) and \( Q \) are sufficiently small for these parameters to be treated as constant.

\( \frac{\Theta^2}{V_0} \) is determined explicitly from equation A4.17 hence \( \lambda \Theta \) can then be obtained. But, since it is \( a \) and not \( \lambda \Theta \) that is used as the parameter for the velocity profile, it is necessary to relate \( a \) to \( \lambda \Theta \).

This is done by eliminating \( \frac{d\Theta^2}{dx} \) from equations A4.10 and A4.11 and results in

\[ \lambda \Theta (H_{12} - 1) = \frac{1}{2} \left( P - \frac{Q}{H_{32}^2} \right) + \lambda \Theta \frac{U}{H_{32}} \frac{dH_{32}}{dU_\infty} \quad \text{A4.18} \]

**Method of solution:**

As it stands equation A4.18 is not in a form suitable for solution. Therefore, using equations A4.13 and A4.14, equation A4.18 is rearranged to:

\[ a = \left\{ \lambda \Theta \left( H_{12} - 1 \right) + \frac{Q}{2H_{32}^2} - \frac{a^2}{105} + \frac{a^3}{252} - \lambda \Theta \frac{U}{H_{32}} \frac{dH_{32}}{dU} \right\} \left[ \frac{35}{4} \right] \quad \text{A4.19} \]

Also \( H_{12} = \frac{D}{E} \); \( H_{32} = \frac{F}{E} \) and \( Q \) are functions of the profile parameter, \( a \), and substitution of these functions, given by equations A4.12, A4.13, A4.14 and A4.16, into equation A4.19 establishes the relationship between \( a \) and \( \lambda \Theta \).
The value of \( \Theta \) and hence \( \lambda = \Theta \frac{d u}{d x} \) is evaluated from step-by-step integration of the quadrature given by equation A4.17. Then, for a first approximation to the solution of equation A4.19 the term \( \lambda \frac{d H_{32}}{d u} \) is neglected and, at each integration step, the value of the profile parameter, \( \alpha \), is determined by iteration of the resulting equation. Using the values of \( \alpha \) determined from the first approximation to the solution of A4.19, a curve fitting routine is employed to estimate the neglected term. This term is then included in the second approximation to the solution of equation A4.19 and so on.

With the profile parameter, \( \alpha \), evaluated at each step, the profile shape factors, \( H_{12} \) and \( H_{32} \) are calculated from:

\[
H_{12} = \frac{D}{E} \quad ; \quad H_{32} = \frac{F}{E}
\]

The values of \( D, E \) and \( F \) being obtained from equations A4.12, A4.13 and A4.14 respectively. The values of momentum thickness, \( \Theta \), obtained from the numerical integration of equation A4.17 are used to calculate the remaining boundary layer integral thicknesses \( C^* \) and \( C^* \) from equations A4.8

The skin friction coefficient, \( C_f \), is obtained from equation A4.9 knowing:

\[
\tau_0 = \mu \left( \frac{d u}{d y} \right)_{y=0}
\]

and

\[
C_f = \frac{2 \tau_0}{\rho u_\infty^2}
\]

then

\[
C_{f^*} = \frac{\rho u_\infty}{\Theta u_\infty}
\]
For a zero pressure gradient flow i.e. $\lambda_0 = 0$ equation A4.18 reduces to:

$$p = \frac{Q}{H_{32}^2}$$

the solution of which is trivial and results in:

$$a = 1.857$$

Other specific values of $a$ which correspond to definite conditions are:

(i) at a separation point; $a = 0$

(ii) at a stagnation point; $a = 4.00$
A4.2 Alber's (1968) method for turbulent boundary layers

In general the boundary layer velocity profile employed in a turbulent boundary layer integral method is represented by a two parameter family of the form:

\[ \frac{U}{U_\infty} = f \left( \tau, \frac{C_f}{y}, \frac{y}{S} \right) \]

For the basic boundary layer problem there are then three unknowns \( S \), \( \tau \) and \( C_f \) and therefore three equations are required to solve for these unknowns. The three equations usually employed are:

1. The momentum integral equation (A4.3)
2. Some local friction law
3. An auxiliary equation

The method of Alber uses the two-parameter formulation of Coles (1956) for the local skin friction law, i.e.

\[ \frac{U}{U_\infty} = \frac{1}{k} \ln \frac{U_{\text{wall}}}{U_\infty} + C + \frac{\tau}{k} \sin^2 \left( \frac{\tau}{2} \cdot \frac{y}{S} \right) \quad \text{A4.20} \]

and the energy integral equation (A4.4) as the auxiliary equation. By setting \( u = U_\infty \) at \( y = S \) and using the wake integration result:

\[ \frac{S^*}{S} = \frac{1 + \tau}{k} \approx \frac{5}{k} \left( 1 + \frac{\tau}{S^*} \right) \quad \text{where} \quad \lambda = \frac{1}{2} \sqrt{\frac{\tau}{S^*}} \]

in A4.20, the following expression which relates \( C_f \) to \( S^* \) and \( \tau \) is obtained:

\[ \frac{U}{U_\infty} = \lambda = \frac{1}{k} \ln \left\{ \frac{k S^*}{1 + \frac{\tau}{S^*}} \right\} + \frac{2 \tau}{k} \quad \text{A4.21} \]

Defining the shape factors:

\[ \lambda = \frac{\Theta}{S^*} \quad \text{A4.22} \quad \lambda = \frac{\Theta}{S^*} \quad \text{A4.23} \]

the momentum integral and energy integral equations are then written in the form:
momentum

\[ \frac{C_f}{2} = \int \frac{d}{d\tau} \left[ \frac{d}{d\tau} \left( \frac{dU}{d\tau} \right) + \left( \frac{1}{X} + 2 \right) \frac{dU}{d\tau} \right] \frac{dU}{d\tau} \quad \text{A4.24} \]

energy

\[ \frac{d}{d\tau} \left[ U^3 \int \frac{dU}{d\tau} \right] = 2 \frac{D}{\rho} \quad \text{A4.25} \]

where

\[ D = \int_0^\infty \tau \left( \frac{dU}{d\tau} \right) \, d\tau \]

Equations A4.24 and A4.25 along with a differential form of the local friction law, A4.21, are then used to obtain the three differential equations needed to describe the development of \( \xi^* \), \( f \) and \( \eta \).

Expanding and rearranging A4.24

\[ X \frac{d\xi^*}{d\tau} + \xi^* \left( \frac{d\xi^*}{d\tau} \right) \frac{d\Pi}{d\tau} + \xi^* \left( \frac{d\Pi}{d\tau} \right) \frac{d\xi^*}{d\tau} = \frac{1}{2} \left( \frac{d\Pi}{d\tau} \right) \frac{d\xi^*}{d\tau} \frac{dU}{d\tau} \quad \text{A4.26} \]

Expanding and rearranging A4.25

\[ J \frac{d\xi^*}{d\tau} + \xi^* \left( \frac{d\xi^*}{d\tau} \right) \frac{d\Pi}{d\tau} + \xi^* \left( \frac{d\Pi}{d\tau} \right) \frac{d\xi^*}{d\tau} = C_D - 3 \frac{J \xi^*}{U^3} \frac{dU}{d\tau} \quad \text{A4.27} \]

where \( C_D = \frac{2D}{\rho U^3} \) and is called the "Dissipation integral".

again \( J = h (\Pi, \xi) \) : \[ J \frac{d\Pi}{d\tau} = \frac{dJ}{d\tau} \]

and therefore the energy equation can be written in the form:-

\[ J \frac{d\xi^*}{d\tau} + \xi^* \left( \frac{d\xi^*}{d\tau} \right) \frac{d\Pi}{d\tau} + \xi^* \left( \frac{d\Pi}{d\tau} \right) \frac{d\xi^*}{d\tau} = C_D - 3 \frac{J \xi^*}{U^3} \frac{dU}{d\tau} \quad \text{A4.27} \]
The final differential equation is obtained by differentiating the local friction law i.e. equation A4.21, and results in:

\[
\frac{d}{dx} \frac{\delta^*}{1 + \eta} + \frac{k \delta^* \frac{d \delta^*}{dx}}{\frac{d \delta^*}{dx}} = \frac{\delta^*}{U_\infty} \frac{dU_\infty}{dx}
\]

A4.28

The only unknown in equations A4.26, A4.27 and A4.28 is now the dissipation integral, \( C_D \).

For the case of turbulent equilibrium flows, i.e. for the condition \( \eta_T = \text{constant} \) and \( \beta_T = \text{constant} \), Alber derives an exact expression for \( C_D \) from equations A4.26 - A4.28 resulting in:

\[
C_D = \left[ 1 + \beta_T \right] \frac{k \frac{\delta^*}{\delta^*}}{\frac{\delta^*}{\delta^*}} - 2 \beta_T \frac{\delta^*}{\delta^*}
\]

A4.29

where \( \beta_T = \frac{\delta^*}{\delta^*} \frac{dP}{d\eta} \) or \( \beta_T = \frac{\delta^*}{\delta^*} \frac{dU_\infty}{dx} \)

The dissipation integral is then 'unhooked' from the pressure gradient parameter \( \beta_T \) by assuming that \( \eta_T \) is uniquely related to \( \beta_T \) for nonequilibrium flows. A convenient curve fit given by White (1974) i.e.:

\[
\beta_T = (1.25 \eta)^{4/3} - 0.5
\]

A4.30

is used in this case.

Using equation A4.30 in A4.29 allows equations A4.26 - A4.28 to be solved for the development of a general non-equilibrium turbulent boundary layer for a given set of initial conditions:

\( C_{f_0}, \delta^* \eta_0 \)

Solution procedure:

To recap, the equations to be solved are:
momentum
\[ \chi \frac{d \xi}{dx} + \xi P \frac{d \eta}{dx} + \xi Q \frac{d \xi}{dx} = f^2 - (2 \chi + 1) \frac{\xi}{u_\infty} \frac{d u_\infty}{dx} \] — A4.31

energy
\[ \int \frac{d \xi}{dx} + \xi R \frac{d \eta}{dx} + \xi S \frac{d \xi}{dx} = C_D - 3 \int \frac{\xi}{u_\infty} \frac{d u_\infty}{dx} \] — A4.32

friction law
\[ \frac{d \xi}{dx} + \xi T \frac{d \eta}{dx} + \frac{k \xi}{\xi^2} \frac{d \xi}{dx} = - \frac{\xi}{u_\infty} \frac{d u_\infty}{dx} \] — A4.33

dissipation integral
\[ C_D = \frac{\left[1 + \beta(\pi + 1)\right] \left[\frac{k \xi}{\xi^2} - 1\right]}{\left[\frac{k \xi}{\xi^2} - \pi\right]} - 2 \int \xi^2 \] — A4.34

with \[ \beta = \left(\frac{1.25 \pi}{4/3}\right) - 0.5 \]

and \[ P = \frac{\partial X}{\partial \pi} ; \quad Q = \frac{\partial X}{\partial \xi} ; \quad R = \frac{\partial X}{\partial \eta} ; \quad S = \frac{\partial X}{\partial \xi} \]

\[ T = \frac{(1 + 2 \pi / \pi)}{(1 + \pi)} \]

The partial derivatives \( P, Q, R, \) and \( S \) which appear in the above equations are replaced by algebraic functions of \( \pi \) and \( \xi \) derived from the wake integrations of A4.20 i.e.

\[ \frac{\xi}{\xi} = \frac{1 + \pi}{k \lambda} \] — A4.35

\[ \frac{\pi}{\xi} = \frac{1 + \pi}{k \lambda} \left(\frac{2 \cdot 3.18 \pi + 1.5 \pi^2}{k^2 \lambda^2}\right) \] — A4.36

\[ \frac{\xi}{\xi} = \frac{3 \pi}{\xi} - \frac{\xi}{\xi} + \frac{6 + 4 \cdot 14 \pi + 8 \cdot 5 \pi^2 + 2 \cdot 56 \pi^3}{k^3 \lambda^3} \] — A4.37

and result in :-
\[
\frac{\partial X}{\partial \pi} = \rho = \frac{1}{(1 + \pi)} \left[ \int \frac{3 \cdot 18 + 3 \pi}{k} + (X - 1) \right] \quad \text{--- A4.38}
\]

\[
\frac{\partial X}{\partial \frac{3}{5}} = Q = \frac{2 + 3 \cdot 18 \pi + 1 \cdot 5 \pi^2}{k(1 + \pi)} = \frac{(X - 1)}{\frac{3}{5}} \quad \text{--- A4.39}
\]

\[
\frac{\partial J}{\partial \pi} = R = \frac{2 - \frac{\pi}{(1 + \pi)} - \frac{\pi}{(1 + \pi)^2} \left[ 3(3 \cdot 18 + 3 \pi) + \frac{f}{k} (11 \cdot 14 + 17 \pi + 7 \cdot 68 \pi^2) \right]}{1 + \pi} \quad \text{--- A4.40}
\]

\[
\frac{\partial J}{\partial \frac{3}{5}} = S = \left[ \frac{J - 2}{\frac{3}{5}} \right] + \frac{f}{k^2} \left( \frac{6 + 11 \cdot 14 \pi + 8 \cdot 5 \pi^2 + 2 \cdot 56 \pi^3}{1 + \pi} \right) \quad \text{--- A4.41}
\]

where

\[
X = \frac{\theta}{\theta^*} = 1 - \int \left( \frac{2 + 3 \cdot 18 \pi + 1 \cdot 5 \pi^2}{k(1 + \pi)} \right)
\]

and

\[
J = \frac{\xi^*}{\xi} = 2 - \frac{3 \cdot 5 \pi}{k(1 + \pi)} + \frac{f^2}{k^2} \left( \frac{6 + 11 \cdot 14 \pi + 8 \cdot 5 \pi^2 + 2 \cdot 56 \pi^3}{1 + \pi} \right)
\]

Solving equations A4.31 - A4.33 for \( \frac{d\pi}{d\pi} \), \( \frac{d\xi}{d\pi} \), and \( \frac{d\xi^*}{d\pi} \) results in, after some manipulation:

\[
\frac{d\pi}{d\pi} = \left\{ \frac{\xi^2 \cdot C \left( \frac{Q - k^2 \pi}{5} \right)}{\left( S - k^2 \pi \right)^2} - \left[ (1 + \pi) - \frac{2J \left( \frac{k^2 \pi}{5} \right)}{S - k^2 \pi} \right] \frac{\xi^*}{\xi} \cdot \frac{d\xi}{d\pi} \right\}
\]

\[
\frac{d\xi}{d\pi} = \left\{ \frac{\xi^2 \cdot C \left( \frac{P - \pi \pi}{R - 3 \pi} \right)}{\left( R - 3 \pi \right)^2} - \left[ (1 + \pi) - \frac{2J \left( \frac{P - \pi \pi}{R - 3 \pi} \right)}{R - 3 \pi} \right] \frac{\xi^*}{\xi} \cdot \frac{d\xi}{d\pi} \right\}
\]

\[
\frac{d\xi^*}{d\pi} = - \left\{ \frac{\xi^*}{\xi} \cdot \frac{d\xi}{d\pi} + \frac{k^2 \xi^*}{\xi^2} \cdot \frac{d\xi}{d\pi} + \xi^\star \cdot \frac{d\pi}{d\pi} \right\}
\]

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Using the wake integration results for $P, Q, R$ and $S$ i.e. equations A4.30 - A4.34 and equations A4.30 and A4.34 then equations A4.42 - A4.44 can be solved for $\Omega, \zeta$ and $\delta^*$ using a Runge-Kutta technique.
APPENDIX 5

Software Listings
APPENDIX 5 Software Listings

Appendix 5 contains programme listings for both the Data Acquisition, Control and Data Reduction Package and the computational Boundary Layer Prediction Package. It also contains listings of the following programmes:

1) TURBLEV - Page 250 - Described in section 3.9
2) IMPROF2 - Page 252 - Described in section 3.9
3) SIGCALC - Page 254 - Used for the calculation of $\sigma$ from an experimental data file containing $V_y$, $x$ data

On the following page a copy of the flow chart for the Data Acquisition, Control and Data Reduction Package, described in Chapter 5, is included. Next to the points where each new programme is 'called' or 'CHAINed' is the relevant page number on which the programme listing can be found within this Appendix.

The programme listings for the computational model (Tani/Alber), described in chapter 7, along with the listings for the graphics package start at page 258 and include

1) IGBLPR1 - Page 258 - Introductory programme to computational package
2) IGBLPR5 - Page 263 - Main programme
3) GRAFPC3 - Page 269 - Graphics programme used to display predictions

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Programme

PROGSEL

This programme is the introductory programme to the Data Acquisition, Control and Data Reduction Package
10 CLOSE#0
20 REM PROGSEL
30 MOD#7
40 PRINTTAB(0,5);CHR$132"DO YOU WANT TO:""* 
50 PRINTTAB(5,10);CHR$132"READ AN EXISTING FILE"
60 PRINTTAB(5,15);CHR$132"CREATE A NEW FILE"
70 VDU 310,24
80 selX=GET
90 IF selX = 49 GOTO 120
100 IF selX = 50 CHAIN"5.4"
110 IF selX<>49 OR selX<>50 GOTO 40
120 CLS
130 PRINTTAB(0,10);CHR$132"DO YOU WANT TO SEE DISK CATALOG"*
140 GOTO 160
150 PRINTTAB(0,10);CHR$132"DO YOU WANT TO SEE ANOTHER CATALOG" * 
160 IF GET$="NJ GOTO 250
170 PRINTTAB(0,15);CHR$131;"WHICH DRIVE"
180 DX=GET
190 PRINTCHR$133;"PRESS SPACE TO CONTINUE"
210 spaceX=GET
220 IF spaceX<>32 GOTO 200
230 CLS
240 GOTO 150
250 CLS
260 PRINTTAB(5,12);CHR$134;"WHICH DRIVE IS FILE ON"*
270 DX=GET
280 PROCdrive(DX)
290 CLS
300 PRINTTAB(5,12);CHR$134;"INPUT NAME OF FILE TO BE READ"*
310 INPUT TAB(16,14);E$
320 fileX=OPENOUT(*DATA*)
330 PRINT!fileX,E$
340 CLOSE# fileX
350 DR.0
360 CHAIN"6.3"*
370 END
380 DEFPROCdriver(DX)
390 IF DX=48 THEN #1.
400 IF DX=49 THEN #1.
410 IF DX=50 THEN #2.
420 IF DX=51 THEN #3.
430 ENDPROC
435 DEFPROCdrive(DX)
440 IF DX=48 THEN #1.
450 IF DX=49 THEN #1.
460 IF DX=50 THEN #2.
470 IF DX=51 THEN #3.
480 ENDPROC
Programme

5.4

Programme 5.4 is the main Data Acquisition and Control programme described in some detail in section 3.9
10 REM PROG 5.4 DATA ACQUISITION & CONTROL PROG
20 MODE 3
30 VDU23,240,195,36,24,24,36,36,36,24
40 *10
50 CLOSE # 0
60 ddr=OPENUP"BUS &C002"
70 BPUT#ddr,0,FF
80 CLOSE # ddr
90 pbX=OPENUP"BUS &C000"
100 BPUT#ddr,1
110 AZ=OPENUP"CU-DACB &C000"
120 PRINTTAB(20,12) "SWITCH ON STEPPER MOTOR & H.W ANEMOMETER"
130 H=INKEY(480)
140 CLS
150 PRINT
160 PRINT
170 DIM Y3(40),B(48),ul(40),yl(40),u(40),y(40),RM3(40),IM3(40)
180 PROC calc()on
190 CLS
200 PRINTTAB(15,12) "INPUT TEMPERATURE IN Deg C"
210 INPUTTAB(40,14) t
220 PRINTTAB(15) "INPUT PRESSURE IN mm Hg"
230 INPUTTAB(40) z
240 PRINT
250 PRINT
260 PRINT
270 PRINTTAB(15,6) "INPUT UPPER STEP INCREMENT" 
280 INPUTTAB(40,8) ST1
290 PRINT
300 PRINTTAB(15) "INPUT LOWER STEP INCREMENT"
310 INPUTTAB(40) ST2
320 PRINT
330 PRINTTAB(15) "INPUT No OF Pts AT LOWER STEP INCREMENT"
340 INPUTTAB(40) P1
350 PRINT
360 PRINTTAB(15) "INPUT Y DATUM IN m"
370 INPUTTAB(40) Ydat
380 PRINT
390 PRINT
400 PRINTTAB(15,12) "INPUT DIST. FROM L.E. IN m"
410 INPUTTAB(40) XI
420 PRINT
430 PRINTTAB(15) "INPUT SPANWISE LOCATION IN m"
440 INPUTTAB(40) Z
450 CLS
460 PRINTTAB(15,12) "NAME OF DATA FILE"
470 INPUTTAB(40) EE#
480 CLS
490 K1 = ST1/YC2
500 K2 = ST2/YC2
510 PTR#AX=0
520 FOR IX=1 TO 5000
530 YDX=BBETI AX
540 YD1X=YD1X+YDX
550 NEXT
560 YD2X=YD1X/100
570 VDU2
580 VDU 1,27,1,69
590 PRINT Velocity Y-Pos. Inte.
600 PRINT
610 VDU 1,27,1,70
620 PRINT
630 FOR G TO 17040
640 Y2=G
650 FOR IX=1 TO 1000
660 Y12=BBETI AX
670 Y22=Y2+Y12
680 NEXT
690 C22=0
700 PTR#AX=2
710 FOR IX=1 TO 5000
720 C12=BBETI AX
730 AX=C22+C12
740 NEXT
750 C3=C22/5000
760 IM12=0
770 PTR#AX=4
780 FOR IX=1 TO 10000
790 IM12=BBETI AX
800 IM12=IM12+IM2
810 NEXT
820 RM12=0
830 PTR#2=6
840 FOR IZ=1TO1000
850 RMZ=BGET#1AZ
860 RMX=RMI+RMZ
870 NEXT
880 RMZ=BGET#1AZ
890 IMZ=INT(RMI+RMZ)
900 RMS(D)=INT(RM2X+RM2X+RMZ)
910 Y3(D)=(Y3(I)-Y3(D))*18.52*YC
920 nX=nX+l
930 IF Y3(B)<0.6 GOTO 1010
940 PTR1AZ=0
950 IF nX>P1X GOTO 1130
960 BPUTipbX,0
970 REPEAT UNTIL BGET#1AZ<(YD2X-nX*K2)
980 BPUTipbX,1
990 nX=nX+l
1000 BPUTipbX,0
1010 REPEAT UNTIL BGET#1AZ<(YD2X-(X+0X*K1))
1020 BPUTipbX,1
1030 BPUTipbX,1
1040 NEXT D
1050 uinf=(u1(Q)+u1(Q-1)+u1(Q-2))/3
1060 BPUT#21,1
1070 close#2=CLOSE#1
1080 FOR i=1TOnX
1090 y(i)=yl(i)/u(i)=INT(u(i)*180+8.51/180)
1100 y(i)=INT(y(i)*100+8.51/100)
1110 IF y(i)<0.2 GOTO 1250
1120 AVIH2=AVIH2+IM3(i)
1130 CtX=CtX+l
1140 PRINTTAB(15);y(i);TAB(29);y(i);TAB(45);y(i);TAB(59);y(i);TAB(68);IM3(i)
1150 NEXT B
1160 VDU 1,27,1,69
1170 PRINT"* n",y (mm) Vel. m/s u/uinf y/d RMS Base"
1180 VDU 1,27,1,70
1190 PRINT
1200 PRINT
1210 PRINT
1220 PRINT
1230 PRINT
1240 PRINT"DIST.FROM L.E.=";X1;"",","SPANWISE LOCATION =";Z;"",",","Vel. i/s u/tiinf y/dRHS Gaea1"
1250 PRINTuinf=INT(uinf*100+8.51/180)
1260 PRINT"FREE STREAM VELOCITY"
1270 PRINTFDU 1,27,1,69
1280 PRINT
1290 PRINT
1300 PRINT
1310 PRINT
1320 PRINT
1330 PRINT
1340 PRINT
1350 PRINT
1360 PRINT"d = INT(d100+0.5)/100"
1370 PRINT"APPROX. EDGE OF BOUNDARY LAYER = ";d;"",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",",","," DIFF 90°"
1668 INPUTTAB(40);EI1M
1672 EI1=EI1M/5
1680 GOTO1700
1692 IFy(1)>0.2 GOTO1758
1700 AVIM3 = AVIM3/CX
1710 AVIM = INT(AVIM3*1000+0.5)/1000
1720 VDU 2
1730 GOTO1760
1740 AVIM=0
1750 PRINT
1760 PRINT
1770 PRINT
1772 PRINT*"EYEBALL AVE OF INTERMITTENCY AT y/d=0.2 = ";EI1M
1780 PRINT
1780 PRINT
1790 PRINT*"AVE. OF INTERMITTENCY VALUES BELOW (y/d=0.2)=";AVIM
1800 VDU 3
1810 *DISK
1820 PRINT*"ON WHICH DRIVE IS DATA TO BE STORED"
1830 GOTO1910
1840 ON ERROR OFF
1850 CLOSE# 0
1860 PRINT*"DISK FULL SELECT DRIVE OTHER THAN DRIVE ";DZ-4B
1870 DZ=SET
1880 PROCdrive(DZ)
1890 PROCfile(u,y,nX,uinf,XI,Z,d,E$,t,z,RH3,IM3,AVIM,EIM)
1900 CLS
1910 CHAIN*6.3*
1920 END
1930 DEFPROCcalcon
1940 CC=7.928E-2
1950 YC=1.98E-2
1960 YC2=10.52*YC
1970 IMC=4.05BC-3
1980 RMC=1.471E-2
1990 ENDPROC
2000 DEFPROCfile(u,y,y1nX1uinf,XI,Z,d,E$,t,z,RH3,IM3,AVIM,EIM)
2010 ON ERROR GOTO1880
2020 X2=OPENOUT("DATA")
2030 PRINT#X2Z,E$
2040 CLOSE# X2X
2050 WX=OPENOUT (E$)
2060 FOR I=1ToOnZ
2070 PRINT#WX,u(I),y(I)
2080 NEXT
2090 FOR I=1ToOnX
2100 PRINT#WX,RM3(I),IM3(I)
2110 NEXT
2120 CLOSE# WX
2130 ENDPROC
2140 DEFPROCdrive(DZ)
2150 IFDZ=4BTHEN*DZ.0
2160 IFDZ=49THEN*DZ.1
2170 IFDZ=50THEN*DZ.2
2180 IFDZ=51THEN*DZ.3
2190 ENDPROC
Programme 6.3

Programme 6.3 is a graphics programme used to display experimental data, from a data file, on axes of $y/y'$ and $u/u_\infty$. 
CLOSE 0
REM PROG 6.3 LAMINAR/TURBULENT GRAPHICS PROGRAM
MODE 1
CLS
PROC drive(DX)
DIM P(102), Q(40)
DIM u(40), v(40), RM(340), IM(40)
VDU 19, 3, 0, 0, 0
VDU 19, 2, 0, 0, 0
MOVE 125, 825
DRAW 125, 125
DRAW 125, 125
PRINTTAB(1, 1); "Y"
PRINTTAB(1, 11); "/"
PRINT(1, 12); "d*160 PRINTTAB(25, 30); "+u/Uinf"
MOVE 125, 125
DRAW 125, 125
DRAM 125, 125
DRAW 125, 125
PRINTTAB(0, 6); "1.0*
PRINT(0, 17); "8.5B
PRINT(1V, 29); "+0.5"
PRINT(37, 29); "1.0"
PRINTTAB(4, 4); "LAMINAR TURBULENT B.L. PROFILES"
PROC的心理
320 VDU 80, 6, 14, 21, 6
VDU 182, 5, 5, 5, 5, 5, 5
colour 130: colour 1
CLS
PRINT z = "%z"
PRINT x = "%x"
PRINT Uinf = "%uinf"
PRINT d = "%d"
rho = (0.46535*z)/(t+273)
au = (1.725+0.004375*t)/10
rho = (0.46535*z)/(t+273)

FOR Yphol = 0 TO d STEP 0.05
    Y = Yphol/d
    X = (2*Y-2*Y^3+Y^4)+(LAM/6)*(Y-3*Y^2+3*Y^3-Y^4)
END FOR

FOR I = 1 TO n
    A = u(I)*1100+125
    B = y(I)*700+125
    HOVE A, B
    PLOT 69, A, B
    PLOT 85, A-B, B-B
END FOR

DATA 0.0, 0.377, 0.66, 0.74, 1.33, 1.11, 1.99
DATA 0.155, 0.338, 0.267, 0.456, 0.333, 0.575
DATA 0.487, 0.681, 0.481, 0.772, 0.555, 0.846
DATA 0.638, 0.982, 0.782, 0.941, 0.748, 0.955
DATA 0.778, 0.982, 0.876, 0.978, 0.962, 0.983
DATA 0.889, 0.988, 0.926, 0.991, 0.963, 0.994
DATA 1.0, 0.997
DATA 1.25, 1.25
MOVE 125, 125
FOR K = 1 TO 41
    XI = I
    Y1 = X1
    XI = I+8.025
    PI(K) = XI*1100+125
    Q1(K) = Y1*700+125
    DRAW PI(K), Q1(K)
END FOR

DRAW P
DRAW 50
GCOL 0, 1
FOR I = 1 TO n
    A = u(I)*1100+125
    B = u(I)*700+125
    MOVE A, B
    PLOT 69, A, B
    PLOT 85, A-B, B-B
END FOR

NEXT I
VDU 26: VDU 31 0, 31
COLOUR 128
B4BPRI NT"IS A PRINT OF GRAPH REQ'D"
B90 IF GET$="N" GOTO 1020
B90 PRINT "IS PROFILE LAMINAR(L), TURBULENT(T) OR TRANSITIONAL(t)"
900 AS = GET$
910 IF AS = "L" GOTO 950
920 IF AS = "T" GOTO 960
930 IF AS = "t" GOTO 970
940 GOTO 970
950 CHAIN "8.3"
960 CHAIN "9.3"
970 PRINT "IS VALUE OF INTERMITTENCY @ y/d=0.2 GREATER THAN(G) OR LESS THAN(L) 0.5"
980 AS = GET$
990 IF AS = "L" THEN CHAIN "11.3L"
1000 IF AS = "G" THEN CHAIN "11.3T"
1010 GOTO 970
1020 END
1030 DEFPROC file
1040 X2=OPENIN ("DATA")
1050 INPUT#X2,E$
1060 CLOSE#X2
1070 W=OPENIN (E$)
1080 INPUT#n,uinf,X1,Z,d,t,t,z,AVIM,EIM
1090 FOR I = 1 TO n
1100 INPUT#u(I),y(I)
1110 NEXT I
1120 FOR I = 1 TO n
1130 INPUT#rms(I),IM3(I)
1140 NEXT I
1150 CLOSE#W
1160 ENDPROC
1170 DEFPROC drive(DX)
1180 IF DX=4B THEN DR.8
1190 IF DX=49 THEN DR. 1
1200 IF DX=5B THEN DR.2
1210 IF DX=51 THEN DR.3
1220 ENDPROC
Programme

10.3

Programme 10.3 is a screen dump programme which enables a hard copy of a graphics display, on the computer monitor, to be obtained from the Epson line printer
REM Hybrid program to dump all graphics MODEs
REM on the EPSON FT printer
DIM SX &FF
pass number=SX
pattern0=SX+1
pattern1=SX+3
pattern4=SX+9
pattern2=SX+11
pattern2=649844100
pattern4=3F80
pattern1=3F26488
pattern2=SX+5
pattern2=&3F268488
pattern1=SX+3
pattern2=&49844108
(pattern2+4)=fcFF6FB966
SX=SX+17
PROClimits
IF NOT graphics THEN PRINT "Not a graphics MODE. Can't dump.":VDU7:END
PROCassemble
REM enable printer, and set linefeed (send ESC A B)
VDU2, 1.27, 1.65, 1.0
VDU clear paper
VDU1.10, 1.10, 1.0
FOR YX=1023 TO 0 STEP-16
REM send bit code (ESC L 192 3 - 960 dots per line or 640 dots for MODE8)
VDU27.1.76.1.n1.1.n2
FOR YX=B TO 127 STEP step size
Xlo=Xlo+YX*?;10B08
pass=6
CALL pixel
NEXT
VDU1 18
NEXT
REM reset linefeed and disable printer
VDU1,27,1,65,1,12,1,12,3
PRINT "DO YOU WANT TO ANALYSE PROFILE"
IF GET* = *L" GOTO 420
IF A* = *T* 60T0 430
IF A* = *T* GOTO 440
CHAIN "8.3"
CHAIN "9.3"
PRINT "IS VALUE OF INTERMITTENCY @ y/d=8.2 GREATER THAN(L) OR LESS THAN(L) 0.5"
AS = GET#
AS = "L" GOTO 490
AS = "T" GOTO 440
AS = "T" GOTO 440
CHAIN "11.3L"
CHAIN "11.3T"
END
DEFPROC limits
DIM user 3
AX=&87
user=USR(&FFF4)
IF iode>5 OR iode=3 THEN graphics=FALSE ELSE graphics=TRUE
mode=user 72
IF mode=5 OR mode=3 THEN graphics=FALSE ELSE graphics=TRUE
IF mode=THEN n1=128:n2=2 ELSE n1=192:n2=3
IF mode=THEN step size=2:pass number=1: pass number=pattern4 MOD 256:pass number=pattern4 DIV 256
IF mode=1 THEN step size=4:pass number=3: pass number=pattern1 MOD 256:pass number=pattern1 DIV 256
IF mode=1 THEN step size=8:pass number=6: pass number=pattern2 MOD 256:pass number=pattern2 DIV 256
ENDPROC
DEFFPROCassemble
osword=&FFFF
oswrch=&FFE
Xlo=SX
Ylo=SX+2
Yhi=SX+3
value=SX+4
byte=SX+5
pass=SX+6
count 4=SX+7
540 SX=SX+8
550 SX=SX+9
FOR opt = 0 TO 2 STEP 2
710 point ldx IXlo MOD 256
720 point ldy IXlo DIV 256
! subroutine to print a character
. printchar lda #1
jsr oswrch
ida byte
jsr oswrch
rts

decrement Y by 4
.dec Y4 sec
ida Ylo
sbc #4
sta Ylo
bcc dec_Yhi
rts
.dec_Yhi dec_Yhi

increment Y by 16
.inc Y16 clc
ida Ylo
adc #16
sta Ylo
bcs inc_Yhi
rts
.inc_Yhi inc_Yhi
rts

to rotate in two bits. Enter with X=pass, Y=colour
.ten_bits lda 1000,Y
.select appropriate byte of pattern
cpx #8
.if pass is 0 rotate
beq rotate_in
.or A
next two bits in
rotate_out ror A
otherwise dump two bytes
ror A
dex
.has X reached 0?
ror A
rotate_out
.if not dump two more
rol byte
.if so next two bits go into byte
ror A
rol byte
rts

to calculate a whole byte
.one_byte jsr point
ldy value
ida pass
and #3
tax
jsr two_bits
jsr dec_Y4
dec count_4
bne one_byte
.if byte incomplete go back
jsr printchar
print the byte
rts
.MAIN PROGRAM

to calculate and print the pattern for one pixel
.pixel lda #4
sta count_4
reset counter
jsr one_byte
jsr inc_Y16
inc pass
ida pass
jsr one_byte
inc pass
amp pass_number
bne pixel
rts
NEXT
ENDPROC
Programme

8.3

Programme 8.3 is used for the reduction of the mean laminar velocity profiles and is described in detail in section 4.2.
10 MODE 3
20 REM PROG B.3 LAMINAR BOUNDARY LAYER ANALYSIS PROG
30 DIM ul(50),u(50),yl(50),uplus(50),eta(50),y(58),e(58)
40 DIM RM3I50),LM3(50),uplus(50),yplus(50)
50 PROCdrive(D7.)
60 PROC-file
70 *DR.0
80 FOR i = 1 TO n
90 ul(i)=u(i)*uinf
100 yl(i)=y(i)*d
110 NEXT i
120 u = 11.725 + 0.004375*t)/10A5
130 rho = (0.46535*2)/(t+273)
140 nu = au/rho
150 REM CALCULATE SHEAR STRESS AND FRICTION COEFF.
160 FOR k = 1 TO n
170 IF u(1)>0.45 THEN 250
180 IF u(k) >= 0.45 THEN 210
190
200 bubs = sues + ul(k)/yl(k)
210 NEXT k
220 t0 = t*5/UR5/L*1000
230 cf = 2*t0/(rho*uinfA2)
240 REM CALCULATE ERROR AND RMS ERROR OF FIT
250 FOR i = 1 TO n
260 eta(i) = (yl(i)/1000)*SBR(uinf*1800/(nu*x))
270 y(i) = INT(y(i)*1000+0.5)/1000
280 u(i) = INT(u(i)*1080+8.5)/1080
290 e(i) = INT(e(i)*1008+8.5)/1008
300 PRINT TAB(10);yl(l);TAB(20);ul(i);TAB(30);y(i);TAB(40);eta(i);TAB(51);e(i)
310 NEXT i
320 PRINT RMS ERROR OF FIT =",erms
330 PRINT
340 PRINT
350 PRINT
360 PRINT
370 PRINT
380 PRINT
390 PRINT
400 PRINT
410 PRINT
420 PRINT
430 PRINT
440 PRINT
450 PRINT
460 PRINT
470 PRINT
480 PRINT
490 PRINT
500 PRINT
510 PRINT
520 PRINT
530 PRINT
540 PRINT
550 PRINT
560 PRINT
570 PRINT
580 PRINT
590 PRINT
600 PRINT
610 PRINT
620 PRINT
630 PRINT
640 PRINT
650 PRINT
660 PRINT
670 PRINT
680 PRINT
690 PRINT
700 NEXT
710 PRINT "RMS ERROR OF FIT =",erms
PRINT "LAMINAR BOUNDARY LAYER PARAMETERS"
PRINT "APPROX EDGE OF BOUNDARY LAYER"
PRINT "DISPLACEMENT THICKNESS"
PRINT "MOMENTUM THICKNESS"
PRINT "ENERGY THICKNESS"
PRINT "SHAPE FACTOR H12"
PRINT "SHAPE FACTOR H32"
PRINT "MOMENTUM TH. REYNOLDS NO."
PRINT "DISPLACEMENT TH. REYNOLDS NO."
PRINT "SKIN FRICTION COEFF."
PRINT "WALL SHEAR STRESS"
PRINT "DO YOU WANT THIS DATA PLOTTED ON"
PRINT "THE UNIVERSAL VELOCITY PROFILE"
IF GET$='Y' GOTO 1100 ELSE 1120
PRINT "DATA*"
INPUT IX2, E$
CLOSE# IX2
CLS
HUD 2
H = OPEN1N E*
= ;d " ma" 
= ;del1" ma" 
= ;theta" ma" 
= ;del2" ma" 
= ;h12 
= ;h32 
= ;Reo 
= ;Rdis 
= ;cf 
= ;t8"N/ma^2"
1660 INPUT W,n,uinf,x,z,d,t,z,AVIH,EIM
1670 FOR i = 1 TO n
1680 INPUT W,u(i),y(i)
1690 NEXT i
1700 FOR i = 1 TO n
1710 INPUT W,RM3(i),IM3(i)
1720 NEXT i
1730 CLOSE# W
1740 ENDFROC
1750 DEF PROC drive(XX)
1760 IF DX = 4B THEN *DR.0
1770 IF DX = 49 THEN *DR.1
1780 IF DX = 50 THEN *DR.2
1790 IF DX = 51 THEN *DR.3
1800 ENDFROC
1810 DEF PROC file(u1,y1,t0,nu,n,E$)
1820 utau = SQR(t0/rho)
1830 FOR i = 1 TO n
1840 uplus(I) = u(I)/utau
1850 yplus(I) = utau*y1(I)/(nu*100)
1860 NEXT i
1870 X$ = OPENOUT*DATA*
1880 PRINT X$, n, E$
1890 FOR i = 1 TO n
1900 PRINT X$, uplus(I), yplus(I)
1910 NEXT i
1920 CLOSE# X$
1930 ENDFROC
Programme

9.3

Programme 9.3 is used for the reduction of the mean turbulent velocity profiles and is described in detail in section 4.3
10 REM PROG 9.3 TURBULENT BOUNDARY LAYER ANALYSIS PROG
20 REM THIS PROG WILL AUTOMATICALLY DELETE THE POINT NEAREST THE WALL IF ytl < 50
30 REM MODE 7
40 DIM y(40),u(40),ul(40),y(40)
50 DIM resid(40),utau(40),e(40),yplus(40),u(40)
60 DIM x(40),ytl(40),RM3(40),IM3(40)
70 PROCdrive(DX)
80 PROCfile
90 PROC
c
100 FOR i = 1 TO n
110 yl(i)=y(i)*d
120 ul(i)=u(i)*uinf
130 NEXT i
140 rho = (0.46535*z)/(t+273)
150 nu = 0.004375*t
160 Rx = INT((uinf * x / (nu*1080))/1800)*1000
170 n = n0000000000
180 FOR k = 2 TO n
190 yl(k-i) = yl(k)
200 ul(k-l) = ul(k)
210 u(k-l) = u(k)
220 y(k-l) = y(k)
230 RM3(k-l) = RM3(k)
240 IM3(k-l) = IM3(k)
250 NEXT k
260 n = n - 1
270 IF ytl < 50 GOTO 200
280 PRINT "Point Nearest Wall Has Been Deleted As ytl < 50"
290 PROCloglaw(ul,yl,y,Rx,uinf)
300 PROCwalint(ytl,Rx,utau,ul,uinf)
310 PROCparint(u,y,n,d)
320 BIH2 = BIH2
330 BIH4 = BIH4
340 BIH6 = BIH6
350 BIH8 = BIH8
360 yplus = yplus
370 Uplus = Uplus
380 Resid. = Resid.
390 Udef. = Udef.
400 VDU 1,27,1,14:PRINTTAB(12)'FILE".E$
410 PRINT
420 PRINT
430 PRINT
440 PRINT
450 PRINT
460 PRINT
470 PRINT
480 PRINT
490 PRINT
500 PRINT
510 PRINT
520 PRINT
530 PRINT
540 PRINT
550 FOR i = 1 TO n
560 PRINT
570 PRINT
580 PRINT
590 PRINT
600 PRINT
610 PRINT
620 PRINT
630 PRINT
640 PRINT
650 PRINT
660 PRINT
670 PRINT
680 PRINT
690 PRINT
700 PRINT
710 PRINT
720 PRINT
730 PRINT
740 PRINT
750 PRINT
760 PRINT
770 PRINT
780 PRINT
800 PRINT
810 PRINT
820 PRINT
830 uplus(i) = INT(uplus(i)*10000+0.5)/10000
840 yplus(i) = INT(yplus(i)*18880+0.51)/10800
850 resid(i) = INT(resid(i)*10000+B.5)/1B00B
860 udef(i) = INT(udef(i)*10000+0.5)/10800
870 PRINT TAB(20):yplus(i);TAB(32):uplus(i);TAB(43)resid(i);TAB(56);udef(i)
880 y(i) = INT(y(i)*1080+0.5)/18B0
890 u(i) = INT(u(i)*1000+0.5)/1000
900 NEXT i
910 PRINT
920 PRINT
930 PRINT
940 PRINT
950 PRINT
960 VDU 1.27,1,69
970 PRINT Y-aa Y/d U/Uinf*
980 VDU 1.27,1.70
990 PRINT
1000 FOR i = 1 TO n
1010 PRINT TAB(20);yl(i);TAB(38):y(i);TAB(58);u(i)
1020 NEXT i
1030 PRINT
1040 del1=INT (del 1*1000+0.5)/1000
1050 del2=INT(del2*18BB+0.5)/1000
1060 theta=INT(theta*1800+0.5)/1000
1070 h12=INT(h12*1000+0.5)/1000
1080 h32=INT(h32*10000+0.5)/1000
1090 Rtheta=INT(Rtheta)
1100 Rdell=INT(Rdell)
1110 utaul=INT(utaul*1000+0.5)/1000
1120 pi=INT(pi*1000+0.5)/1000
1130 t8=INT(t8*100B+8.55)/1000
1140 PRINT
1150 PRINT
1160 PRINT
1170 PRINT
1180 PRINT*TURBULENT BOUNDARY LAYER PARAMETERS*
1190 PRINT
1200 PRINT
1210 PRINT"DISPLACEMENT THICKNESS" ;";del1" ma*
1220 PRINT
1230 PRINT"MOMENTUM THICKNESS" ;";theta" mm*
1240 PRINT
1250 PRINT"ENERGY THICKNESS" ;";del2" ma*
1260 PRINT
1270 PRINT"SHAPE FACTOR H12" ;";h12" mm*
1280 PRINT
1290 PRINT"SHAPE FACTOR H32" ;";h32*
1300 PRINT
1310 PRINT*MOM. TH. REYNOLDS No. ;";Rtheta*
1320 PRINT
1330 PRINT*DISP. TH. REYNOLDS No. ;";Rdel1*
1340 PRINT
1350 PRINT* cf (LUD/TILL) ;";cfl*
1360 PRINT* cf (LOG-PLT) ;";cf2*
1370 PRINT* cf (COLES-FORM) ;";cf3*
1380 PRINT
1390 PRINT"WALL FRICTION VELOCITY" ;";utau1" m/s*
1400 PRINT
1410 PRINT"WALL SHEAR STRESS" ;";t0" N/m^2*
1420 PRINT
1430 PRINT"MAKE PARAMETER P1" ;";pi*
1440 VDU 3
1450 PRINT
1460 PRINT
1470 PRINT
1480 PRINT"DO YOU WANT A PLOT OF DATA ON UNIVERSAL*"
1490 PRINT"TURBULENT B.L. VELOCITY PROFILE*"
1500 AS = GET$ 5
1510 IF AS = "Y" GOTO 1520 ELSE 1540
1520 PROCPlfile(uplus,yplus,n,E$)
1530 CHAIN "TUGRAF2$
1540 END
1550 DEFPROCloglaw(ul,y1,y,n,nu,uinf)
1560 FOR k = 1 TO n
1570 utau(k) = 1
1570 GOTO 1600
1580 utau(k) = itut(k)
1590 yplus(k) = utau(k)*y1(k)/(nu*uinf)
1600 e(k) = utau(k) - itut(k)
1610 IF ABS(e(k))<0.00001 THEN 1650
1620 IF 0.05 < 0.2 THEN 1650
1630 GOTO 1590
1640 IF yplus(k) <= 30 OR y(k) >= 0.2 GOTO 1600
1660 1 = 1 + 1
1670 sum = sum + utaul(k)
1680 NEXT k
1690 utaul = sum/n
1700 FOR i = 1 TO n
1710 uplus(i) = u(i)/utaul
1720 yplus(i) = utaul*(y(i))/(nu*100)
1730 resid(i) = uplus(i) - (2.439*LN(yplus(i))*5.2)
1740 NEXT i
1750 NEXT i
1760 vtl = yplus(3)
1770 ENDPROC

1780 DEFPRQCwalint(ypl,nu,utaul,uinf)
1790 cl = 540.6
1800 c2 = 654.6
1810 c3 = 82778.0
1820 a = 2.439
1830 b = 5.2
1840 p11 = a*(ypl*(LN(ypl)**1)-50*(LN(50)**1))
1850 p12 = b*(ypl*50)
1860 intlw = p11 + p12
1870 p21 = ypl*(LN(ypl)**2-50*(LN(50)**1)**2
1880 p22 = -2*(ypl*(LN(ypl)**1)-50*(LN(50)**1))
1890 p23 = 2*b*a*(ypl*(LN(ypl)**1)-50*(LN(50)**1))
1900 p24 = b**2*(ypl*50)
1910 p31 = ypl*(LN(ypl)**3-3*ypl**2*(LN(ypl)**1)**2
1920 p32 = -50*(LN(50)**2)**2-6*ypl*(LN(50)**1)**2
1930 p33 = ypl*(LN(ypl)**1)-50*(LN(50)**1)**2
1940 p34 = 2*(ypl*(LN(ypl)**1)-50*(LN(50)**1))
1950 p35 = 3*a*b**2*(ypl*(LN(ypl)**1)-50*(LN(50)**1))
1960 p36 = b**3*(ypl*50)
1970 intsw = a**2*(p31 + p32) + 3*b*a**2*(p33 + p34) + p35 + p36
1980 s1 = (cl + intlw)/uinf
1990 s2 = (ypl*(nu/utaul)*1000) - s1
2000 s3 = (c2 + int2w)/(nu/uinf)*(utaul/uinf)*1000
2010 s4 = (c3 + int3w)*uinf/uinf*(utaul/uinf)**2*1000
2020 ENDPRQC

2030 DEFPRQCparint(u,y,n,d)
2040 DIN a(30)-b(30),c(30),det(30)
2050 DIN inti(30),ini2(30),int3(30),int4(30)
2060 DIN int5(30),int6(30),int7(30),into(30)
2070 n2 = n - 2
2080 FOR k = 3 TO n2
2090 det1 = 1*(y(k+1)**2-y(k+2)**2-y(k+1)**2)
2100 det2 = y(k)**2*(y(k+1)**2-y(k)**2)
2110 det3 = y(k)**2*(y(k+1)-y(k))
2120 det(k) = det1 - det2 + det3
2130 a1 = y(k+1)**2*(y(k+2)**2-y(k+1)**2)
2140 a2 = y(k)**2*(y(k+2)**2-y(k+1)**2)
2150 a3 = y(k)**2*(y(k+2)-y(k+1))
2160 b1 = y(k)**2*(y(k+2)-y(k+1))
2170 a(k) = (a1 - a2 + a3)/det(k)
2180 b(k) = (a1 - a2 + a3)/det(k)
2190 c(k) = (a1 - a2 + a3)/det(k)
2200 pi11 = a(k)*(y(k+1)-y(k))+b(k)*(y(k+2)**2-y(k+1)**2)
2210 pi12 = c(k)*(y(k+2)**3-y(k+1)**3)
2220 pi21 = a(k)*(y(k+2)-y(k+1))-b(k)*(y(k+2)**2-y(k+1)**2)
2230 pi22 = c(k)*(y(k+2)**3-y(k+1)**3)
2240 pi31 = a(k)*(y(k+2)-y(k+1))-b(k)*(y(k+2)**2-y(k+1)**2)
2250 pi32 = c(k)*(y(k+2)**3-y(k+1)**3)
2260 pi41 = a(k)*(y(k+2)-y(k+1))-b(k)*(y(k+2)**2-y(k+1)**2)
2270 pi42 = c(k)*(y(k+2)**3-y(k+1)**3)
2280 pi51 = a(k)+(y(k+1)-y(k))
2290 pi52 = a(k)+(y(k+1)-y(k))
2300 pi53 = (2*a(k)+(y(k+1)-y(k))**3-y(k)**3)/3
2310 pi54 = b(k)**4-(y(k)**4)/2
2320 pi55 = c(k)*(y(k+1)**5-y(k)**5)/5
2330 into = into + 1
2340 int5(k) = p51+p52+p53+p54+p55
2350 int6(k) = p51+p52+p53+p54+p55
2360 int7(k) = p51+p52+p53+p54+p55
2370 int8(k) = p51+p52+p53+p54+p55
2380 int9(k) = p51+p52+p53+p54+p55
2390 ENDPRQC
\[
\begin{align*}
\text{int}_b(k) &= \text{pi}_b1 + \text{pi}_b2 + \text{pi}_b3 + \text{pi}_b4 + \text{pi}_b5 \\
\text{pi}_b1 &= a(k) * a(k) * a(k) * (y(k+1) - y(k)) \\
\text{pi}_b2 &= 3 * a(k) * a(k) * b(k) * (y(k+1) - y(k)) \\
\text{pi}_b3 &= a(k) * a(k) * c(k) * (y(k+1) - y(k)) \\
\text{pi}_b4 &= b(k) * a(k) * (y(k+2) - y(k)) \\
\text{pi}_b5 &= 0.6 * c(k) * a(k) * (y(k+2) - y(k)) \\
\text{pi}_b6 &= b(k) * c(k) * (y(k+2) - y(k)) \\
\text{pi}_b7 &= c(k) * c(k) * (y(k+2) - y(k)) \\
\ln_1(k) &= \text{pi}_b1 + \text{pi}_b2 + \text{pi}_b3 + \text{pi}_b4 + \text{pi}_b5 + \text{pi}_b6 + \text{pi}_b7 \\
\text{pi}_b8 &= a(k) * a(k) * (y(k+2) - y(k)) \\
\text{pi}_b9 &= 3 * a(k) * a(k) * b(k) * (y(k+2) - y(k)) \\
\text{pi}_{b10} &= a(k) * a(k) * c(k) * (y(k+2) - y(k)) \\
\text{pi}_{b11} &= b(k) * a(k) * (y(k+3) - y(k)) \\
\text{pi}_{b12} &= 0.6 * c(k) * a(k) * (y(k+3) - y(k)) \\
\text{pi}_{b13} &= b(k) * c(k) * (y(k+3) - y(k)) \\
\text{pi}_{b14} &= c(k) * c(k) * (y(k+3) - y(k)) \\
\ln_2(k) &= \text{pi}_b8 + \text{pi}_b9 + \text{pi}_{b10} + \text{pi}_{b11} + \text{pi}_{b12} + \text{pi}_{b13} + \text{pi}_{b14} \\
\text{pi}_{b15} &= a(k) * a(k) * (y(k+3) - y(k)) \\
\text{pi}_{b16} &= 3 * a(k) * a(k) * b(k) * (y(k+3) - y(k)) \\
\text{pi}_{b17} &= a(k) * a(k) * c(k) * (y(k+3) - y(k)) \\
\text{pi}_{b18} &= b(k) * a(k) * (y(k+4) - y(k)) \\
\text{pi}_{b19} &= 0.6 * c(k) * a(k) * (y(k+4) - y(k)) \\
\text{pi}_{b20} &= b(k) * c(k) * (y(k+4) - y(k)) \\
\text{pi}_{b21} &= c(k) * c(k) * (y(k+4) - y(k)) \\
\ln_3(k) &= \text{pi}_{b15} + \text{pi}_{b16} + \text{pi}_{b17} + \text{pi}_{b18} + \text{pi}_{b19} + \text{pi}_{b20} + \text{pi}_{b21} \\
\text{pi}_{b22} &= a(k) * a(k) * (y(k+4) - y(k)) \\
\text{pi}_{b23} &= 3 * a(k) * a(k) * b(k) * (y(k+4) - y(k)) \\
\text{pi}_{b24} &= a(k) * a(k) * c(k) * (y(k+4) - y(k)) \\
\text{pi}_{b25} &= b(k) * a(k) * (y(k+5) - y(k)) \\
\text{pi}_{b26} &= 0.6 * c(k) * a(k) * (y(k+5) - y(k)) \\
\text{pi}_{b27} &= b(k) * c(k) * (y(k+5) - y(k)) \\
\text{pi}_{b28} &= c(k) * c(k) * (y(k+5) - y(k)) \\
\ln_4(k) &= \text{pi}_{b22} + \text{pi}_{b23} + \text{pi}_{b24} + \text{pi}_{b25} + \text{pi}_{b26} + \text{pi}_{b27} + \text{pi}_{b28} \\
\text{pi}_{b29} &= a(k) * a(k) * (y(k+5) - y(k)) \\
\text{pi}_{b30} &= 3 * a(k) * a(k) * b(k) * (y(k+5) - y(k)) \\
\text{pi}_{b31} &= a(k) * a(k) * c(k) * (y(k+5) - y(k)) \\
\text{pi}_{b32} &= b(k) * a(k) * (y(k+6) - y(k)) \\
\text{pi}_{b33} &= 0.6 * c(k) * a(k) * (y(k+6) - y(k)) \\
\text{pi}_{b34} &= b(k) * c(k) * (y(k+6) - y(k)) \\
\text{pi}_{b35} &= c(k) * c(k) * (y(k+6) - y(k)) \\
\ln_5(k) &= \text{pi}_{b29} + \text{pi}_{b30} + \text{pi}_{b31} + \text{pi}_{b32} + \text{pi}_{b33} + \text{pi}_{b34} + \text{pi}_{b35} \\
\text{pi}_{b36} &= a(k) * a(k) * (y(k+6) - y(k)) \\
\text{pi}_{b37} &= 3 * a(k) * a(k) * b(k) * (y(k+6) - y(k)) \\
\text{pi}_{b38} &= a(k) * a(k) * c(k) * (y(k+6) - y(k)) \\
\text{pi}_{b39} &= b(k) * a(k) * (y(k+7) - y(k)) \\
\text{pi}_{b40} &= 0.6 * c(k) * a(k) * (y(k+7) - y(k)) \\
\text{pi}_{b41} &= b(k) * c(k) * (y(k+7) - y(k)) \\
\text{pi}_{b42} &= c(k) * c(k) * (y(k+7) - y(k)) \\
\ln_6(k) &= \text{pi}_{b36} + \text{pi}_{b37} + \text{pi}_{b38} + \text{pi}_{b39} + \text{pi}_{b40} + \text{pi}_{b41} + \text{pi}_{b42} \\
\text{pi}_{b43} &= a(k) * a(k) * (y(k+7) - y(k)) \\
\text{pi}_{b44} &= 3 * a(k) * a(k) * b(k) * (y(k+7) - y(k)) \\
\text{pi}_{b45} &= a(k) * a(k) * c(k) * (y(k+7) - y(k)) \\
\text{pi}_{b46} &= b(k) * a(k) * (y(k+8) - y(k)) \\
\text{pi}_{b47} &= 0.6 * c(k) * a(k) * (y(k+8) - y(k)) \\
\text{pi}_{b48} &= b(k) * c(k) * (y(k+8) - y(k)) \\
\text{pi}_{b49} &= c(k) * c(k) * (y(k+8) - y(k)) \\
\ln_7(k) &= \text{pi}_{b43} + \text{pi}_{b44} + \text{pi}_{b45} + \text{pi}_{b46} + \text{pi}_{b47} + \text{pi}_{b48} + \text{pi}_{b49} \\
\text{pi}_{b50} &= a(k) * a(k) * (y(k+8) - y(k)) \\
\text{pi}_{b51} &= 3 * a(k) * a(k) * b(k) * (y(k+8) - y(k)) \\
\text{pi}_{b52} &= a(k) * a(k) * c(k) * (y(k+8) - y(k)) \\
\text{pi}_{b53} &= b(k) * a(k) * (y(k+9) - y(k)) \\
\text{pi}_{b54} &= 0.6 * c(k) * a(k) * (y(k+9) - y(k)) \\
\text{pi}_{b55} &= b(k) * c(k) * (y(k+9) - y(k)) \\
\text{pi}_{b56} &= c(k) * c(k) * (y(k+9) - y(k)) \\
\ln_8(k) &= \text{pi}_{b50} + \text{pi}_{b51} + \text{pi}_{b52} + \text{pi}_{b53} + \text{pi}_{b54} + \text{pi}_{b55} + \text{pi}_{b56} \\
\end{align*}
\]

\[
\begin{align*}
\text{pi}_{b57} &= a(k) * a(k) * (y(k+9) - y(k)) \\
\text{pi}_{b58} &= 3 * a(k) * a(k) * b(k) * (y(k+9) - y(k)) \\
\text{pi}_{b59} &= a(k) * a(k) * c(k) * (y(k+9) - y(k)) \\
\text{pi}_{b60} &= b(k) * a(k) * (y(k+10) - y(k)) \\
\text{pi}_{b61} &= 0.6 * c(k) * a(k) * (y(k+10) - y(k)) \\
\text{pi}_{b62} &= b(k) * c(k) * (y(k+10) - y(k)) \\
\text{pi}_{b63} &= c(k) * c(k) * (y(k+10) - y(k)) \\
\ln_9(k) &= \text{pi}_{b57} + \text{pi}_{b58} + \text{pi}_{b59} + \text{pi}_{b60} + \text{pi}_{b61} + \text{pi}_{b62} + \text{pi}_{b63} \\
\end{align*}
\]
Programme

**TUGRAF2**

Programme TUGRAF2 is a graphics programme used for displaying experimental data on the Universal turbulent boundary layer velocity profile. i.e. on axes of $U^+ \sqrt{\nu^+ y^+}$.
10 REM GRAPHICS PROG.*TURGRAF2* _ UNIVERSAL TURBULENT VELOCITY PROFILE PLOT*
20 DIM uplus(40),Lyplus(40)
30 DIM yplus(40)
40 MODE 1
50 PROCCTile
60 VDU 19,3,0,0,0
70 VDU 19,2,4,0,0,0
80 MOVE 125,825
90 DRAW 125,125
100 DRAW 1725,125
110 PRINTTAB(8,2)*TURBULENT BOUNDARY LAYER*
120 PRINTTAB(7,4)*UNIVERSAL VELOCITY PROFILE*
130 PRINTTAB(15,7);*FILE *;E$
140 PRINTTAB(14,14)*U*
150 PRINTTAB(29,30)*Ln Y*
160 PRINT TAB(0,10)*20*
170 PRINTTAB(0,19)*18*
180 PRINTTAB(35,29)*6*
190 PRINTTAB(25,29)*4*
200 PRINTTAB(14,29)*2*
210 PRINTTAB(2,28)*0*
220 MOVE 125,125
230 FOR Uplus = 1 TO 12.5 STEP 0.5
240 Lyplus=LN(Uplus)
250 U1 = Uplus*28+125
260 Y1 = Lyplus*169+125
270 DRAW Y1,U1
280 NEXT
290 MOVE 471,405
300 FOR Uplus =10 TO 600 STEP 50
310 Lyplus = (Uplus-5.0)/2.44
320 U1 = Uplus+28+125
330 Y1 = Lyplus+169+125
340 DRAW Y1,U1
350 NEXT
360 MOVE1139,125;DRAW1139,100
370 MOVE801,125;DRAW801,100
380 MOVE463,125;DRAW463,100
390 MOVE125,685;DRAW125,685
400 MOVE125,405;DRAW125,405
410 RCOL 0,1
420 FOR I = 1 TO n
430 Lyplus(I)=LN(yplus(I))
440 U=uplus(I)*28+125
450 A=Lyplus(I)*169+125
460 MOVE A,B
470 PLOT 69,A,B
480 PLOT 69,A-B,A-B
490 PLOT 85,A+B,A+B
500 NEXT I
510 Print=INKEY(1500)
520 IF Print=32 GOTO530 ELSE 540
530 *RUN MCEUDP*
540 END
550 DEFPROCCTile
560 I3=OPENIN"DATA1*
570 INPUT I3,n,E$
580 FOR I = 1 TO n
590 INPUT I3,uplus(I),yplus(I)
600 NEXT I
610 CLOSE I3
620 ENDP0C
Programme

TURBLEV

Programme TURBLEV is a data acquisition programme used to obtain the streamwise freestream turbulence distribution.
IB HODER
28 REM PROGRAM *TURBLEV* USED TO FIND FREESTREAM
38 REM TURBULENCE LEVEL
48 *10
58 CLOSE#0
68 AX=OPENUP"CU-DACB &C000"
78 CLS
88 PRINTTAB(1,12)CHR$131"INPUT DIST. FROM L.E. IN mm"
98 INPUTTAB(32,12) X
108 CLS
118 PROCcalcon
128 CLS
138 PRINTTAB(1,10)CHR$134"PLEASE WAIT VALUES ARE BEING"
148 PRINTTAB(1,12)CHR$134"CALCULATED"
158 RMSX=0
168 PTRAX=6
178 FOR I=1TO1000
188 RMSX=RMSS+RMSX
198 NEXT
208 VELX=0
218 PTRAX=2
228 FOR I=1TO1000
238 VELX=VELX+VEL2
248 NEXT
258 VELX=VELX+VELX
268 FD=VELX*VC/1000
278 RMSEL=RMSSX+RMST
288 CLS
298 VDU2
308 PRINTTAB(1,6)CHR$132"DISTANCE FROM L.E. = *:X* mm"
318 PRINT:PRINT:PRINT:PRINT
328 PRINTTAB(10)CHR$131"1. 0.01"
338 PRINT:PRINT:PRINT:PRINT
348 PRINTTAB(10)CHR$131"2. 0.03"
358 PRINT:PRINT:PRINT:PRINT
368 PRINTTAB(10)CHR$131"3. 0.1"
378 PRINT:PRINT:PRINT:PRINT
388 PRINTTAB(10)CHR$131"4. 0.3"
398 PRINT:PRINT:PRINT:PRINT
408 FD=GET
418 reqX=AX-48
428 QNREQGOTQ 510,530,550,570
438 RMC=4.902E-4
448 GOTO 590
458 GOTO 590
468 ENDPROC
Programme IMPROF2
Programme IMPROF2 is a data acquisition Programme used to obtain the streamwise 'Near Wall' intermittency distribution. This data is stored in a data file in the form $\bar{\gamma}, x$.
10 REM PROG *IMPROF2* FOR READING IN
20 REM INTERMITTENCY LEVELS
30 DIM EIM(70),AVIM(70),X(78),Z(78)
40 MODE7
50 *10
60 CLOSE#0
70 $2=OPENUP"CU-DAC8 LCO00"
80 $1=1
90 GOTO 120
100 CLS:SUMX=0
110 $1=1+1
120 PRINTTAB(1,10)CHR$130"INPUT DIST FROM L.E. IN mm"
130 INPUTTAB(30,10) X(I)
140 PRINTTAB(1,14)CHR$130"INPUT SPANWISE POS. IN mm"
150 INPUTTAB(30,14) Z(I)
160 CLS
170 PRINTTAB(1,10)CHR$129"WAIT WHILE INTERMITTENCY VALUES"
180 PRINT
190 PRINTTAB(8,12)CHR$129"ARE BEING READ IN"
200 PTRa:=4
210 FOR KX=1 TO 10000
220 IMX=BETiAX
230 SUMX=SUMX+IMX
240 NEXT KX
250 AVIM(I)=((SUMX/10000)-1)*3.9B4E-3
260 CLS
270 PRINTTAB(8,10)CHR$129" INPUT EYEBALL VALUE"
280 PRINTTAB(0,11)CHR$129"
290 PRINTTAB(1,12)CHR$133"OF INTERMITTENCY Volts"
300 INPUTTAB(28,11) EVIM
310 EIM(I)=EVIM*2/10
320 CLS
330 PRINTTAB(0,10)CHR$129"MOVE PROBE TO NEXT POINT THEN"
340 PRINTTAB(0,12)CHR$129"PRESS RETURN"
350 PRINTTAB(0,14)CHR$130"IF RUN IS COMPLETE PRESS (C)"
360 B=GET
370 IF B=67 GOTO 380 ELSE 390
380 CLS
390 END
Programme

SIGCALC

Programme SIGCALC is used to calculate the value of $\gamma$ from experimental $\gamma, \chi$ data obtained using programme IMPROF2.
REM PROG SIGNCALC
10 DIM X(30), AVIM(30), EIM(30), XI(30)
20 DIM 6(30), etal(60)
30 MODE 7
40 CLS
50 IF HX=8GOTO120
60 PRINTTAB(1,8)CHR$134"INPUT NAME OF FILE TO BE READ"
70 PRINTTAB(0,12)CHR$134
80 INPUTTAB(11,12) NAME$
90 PRINTTAB(4,16)CHR$134"WHICH DRIVE IS FILE ON?"
100 DI=SET
110 PROCdrive(DI)
120 PROCfilread(NAME$)
130 CLS
140 PRINTTAB(0,12)CHR$134"DO YOU WANT TO USE"
150 PRINTTAB(5,11)CHR$130"A. EYEBALL VALUES"
160 PRINTTAB(5,14)CHR$130"B. ADC VALUES"
170 PRINTTAB(5,17)CHR$130"C. BOTH EYE & ADC VALUES"
180 QuX=6ET
190 IF QuX=65 GOTO230
200 IF QuX=66 GOTO250
210 IF QuX=67 GOTO270
220 PROCeyeball
230 GOTO280
240 PROCadc
250 GOTO280
260 PROCboth
270 PROCleastsq(K)
280 MODE 7
290 PROClomplotGvX(BuX)
300 M O DE 1
310 PRINTTAB(5,10)CHR$138"WAIT WHILE SIGMA(mean)"
320 PRINTTAB(6,13)CHR$132"IS BEING CALCULATED"
330 PROCsigma(BuX)
340 CLS
350 PRINTTAB(4,IB)CHR$131"MEAN VALUE OF SIGMA = *:INT(AveSig)
360 PRINTTAB(6,14)CHR$132"PRES5 SPACE TO CONTINUE"
370 B=GET
380 PROCfileput
390 *DR.0
400 CHAIN"Plot&V*"e"
410 DEFPROCdrive(DI)
420 #0.
430 IF DX=4B THEN #DR.0
440 IF DX=49 THEN #DR.1
450 IF DX=50 THEN #DR.2
460 IF DX=51 THEN #DR.3
470 ENDPROC
480 DEFPROCfilread(NAME$)
490 IF X<9B50TO570
500 INPUT#fileX,NAMEf
510 CLOSE# fileX
520 EXPDI=OPENIN(NAHE$)
530 INPUT#ExpDI,N,Tu,UB
540 FOR 1=1 TO N
550 INPUT#ExpDI,X(I),EIM(I),AVIM(I)
560 NEXT
570 ENDPROC
580 DEFPROCeyeball
590 K=8
600 FOR I=1 TO N
610 IF EIM(I)<=0.25 OR EIM(I)>=0.75 GOTO720
620 XI(K)=X(I)
630 G(K)=EIM(I)
640 NEXT
650 ENDPROC
660 DEFPROCadc
670 K=0
680 FOR I=1 TO N
690 IF AVIM(I)<=0.25 OR AVIM(I)>=0.75 GOTO870
700 XI(K)=X(I)
710 G(K)=AVIM(I)
720 NEXT
730 ENDPROC
740 DEFPROC both
750 K=0
760 FOR I=1 TO N
770 IF AVIM(I)<=0.25 OR AVIM(I)>=0.75 GOTO810
780 XI(K)=X(I)
790 G(K)=AVIM(I)
800 NEXT
810 ENDPROC
820 ENDPROC
830 DEFPROC both
840 K=0
850 FOR I=1 TO N
860 IF EIH(I) <= 0.25 OR EIH(I) >= 0.75 GOTO 900
870 K=K+1
880 B(K)=EIH(I)
890 XI(K)=X(I)
900 IF AVIH(I) <= 0.25 OR AVIH(I) >= 0.75 GOTO 940
910 K=K+1
920 B(K)=AVIH(I)
930 XI(K)=X(I)
940 NEXT
950 ENDPROC
960 DEFPROC leastsq(K)
970 FOR J=1 TO K
980 SG=SG+B(J)
990 SX=SX+X(J)
1000 SX2=SX2+X(J)^2
1010 S6X=S6X+X(J)*X(J)
1020 NEXT
1030 DEL=K*SX2-SX*SX
1040 H=(K*S6X-SG*SX)/DEL
1050 Const=(SX2*S6-SX*S6X)/DEL
1060 ENDPROC
1070 DEFPROC plotGvX(fluX)
1080 Ct=0
1090 VDU 19,3,3,0,0,0
1100 VDU 19,2,2.0,0,0
1110 MOVE200,700:DRAW200,200:DRAW1200,200
1120 IF fluX=65 GOTO 1200
1130 FOR I=1 TO N
1140 AVIH=AVIH(I)*700+200
1150 XP=(X(I)-X(1))/1.5+200
1160 6COL0.2
1170 PLOT69,XP,AVIH
1180 NEXT
1190 IF fluX=67 GOTO 1200 ELSE 1260
1200 FOR I=1 TO N
1210 EIH=EIH(I)*700+200
1220 XP=(X(I)-X(1))/1.5+200
1230 6CDL0.1
1240 PLOT69,XP,EIH
1250 NEXT
1260 SCOL0.3
1270 FOR G=0.25 TO 0.75 STEP 0.01
1280 XLS=(G-Const)/M
1290 GP=G*700+200
1300 XLSF=(XLS-X(1))/1.5+200
1310 IF GTO I GOTO 1350
1320 MOVE XLSF,GP
1330 Ct=1
1340 NEXT
1350 DRAW XLSF,GP
1360 NEXT
1370 XBAR=(0.5-Const)/M
1380 XBARF=(XBAR-X(1))/1.5+200
1390 SCOL0.1
1400 MOVE XBARF,150:DRAW XBARF,900
1410 MOVE 150,550:DRAW 1200,550
1420 VDU5
1430 MOVE150,130;100:PRINT 'XBAR=';INT(XBAR)
1440 MOVE50,550:PRINT '*G**'
1450 SCOL0.3
1460 MOVE150,700:PRINT '*M':MOVE150,650:PRINT 'A'
1470 MOVE150,620:PRINT '*M':MOVE150,700:PRINT 'A'
1480 MOVE1100,150:PRINT 'X m'
1490 VDU4
1500 ENDPROC
1510 DEFPROC sigama(QuX)
1520 E=8
1530 IF QuX=65 GOTO 1670
1540 FOR I=1 TO N
1550 Inc=1:etax1=-2.5:etax2=2.5
1560 GOTO 1590
1570 etax1=etax1+Inc:etax2=etax1+Inc/10
1580 IF ABS(Gcal-AVIH(I)) <= 0.01 GOTO 1590
1590 FOR etax=etax1 TO etax2 STEP Inc
1600 PROCpolyeta(eta)
1610 IF Gcal>AVIH(I) GOTO 1570
1620 NEXT
1630 etax(I)=eta
1640 NEXT
1650 IF QuX=67 GOTO 1660 ELSE 1790
1790 ENDPROC
1660   K=N
1670   FOR I=1 TO N
1680   K=K+1
1690   etax1=(eta-Inc): eta2x=eta: Inc=Inc/10
1700   IF ABS(Gcal-EIM(I))<0.01 GOTO 1770
1710   FOR etax1 TO etax2 STEP Inc
1720   PROCpolyeta(eta)
1730   IF Gcal>EIM(I) GOTO 1770
1740   NEXT etax1
1750   eta(K)=eta
1760 NEXT I
1770 END PROC

1780   IF Gcal<67 THEN L=2*N ELSE L=N
1790   SUM Sig=0: Ct2X=0
1800   FOR I=1 TO L
1810   IF I>N THEN K=I-N ELSE K=I
1820   IF etal(I)>-2.25 AND etal(I)<-0.25 GOTO 1850 ELSE 1840
1830   IF etal(I)>0.25 AND etal(I)>2.25 GOTO 1850 ELSE 1840
1840   Sigaa=IX(K)-XBAR)/etal(I)
1850   SUM Sig=Sigaa+SUM Sig
1860   CT2X=CT2X+1
1870 NEXT
1880   AVG Sig=SUM Sig/CT2X
1890 END PROC

1900   DEFPROC polyeta(eta)
1910   Heta=SQR(eta*A)
1920   IF eta=8 GOTO 2000
1930   C1=0.8273*Meta
1940   C2=0.094*Heta*A2
1950   C3=0.073*Heta*3
1960   C4=0.0165*Meta*4
1970   6cal=0.5*(1+(eta/(Meta))*(C1-C2-C3+C4))
1980 GOTO 2010
1990 END PROC
2000   Gcal=0.5
2010   END PROC
2020   DEFPROC fileput
2030   fileX=OPENOUT,,GveData"
2040   PRINT# fileX,N,XBAR, AVG Sig, Gxx
2050   FOR I=1 TO N
2060   PRINT# fileX,X(I),AVIH(I),EIM(I)
2070 NEXT
2080   CLOSE# fileX
2090   END PROC
Programme

**IGBLPR1**

Programme IGBLPR1 is the introductory programme for the Tani/Alber computational model described in Chapter 7. This programme is used to read in the initial input data and to estimate the \( \frac{U_0}{H_{12}} \frac{dH_{12}}{dU_0} \) term required for the laminar boundary layer calculation by Tani's method.
10 REM PROG IGBLPR1 USED TO ESTIMATE DGDU TERM FOR TANI'S METHOD
20 MODE7
30 CLS
40 PRINT:PRINT:PRINT:PRINT
50 PRINTCHR$130"ATMOSPHERIC PRESSURE mmHg"
60 PRINT
70 INPUT TAB(15) \, 
80 PRINTCHR$130"AIR TEMPERATURE Deg C"
90 PRINT
100 INPUT TAB(15) t 
110 PRINTCHR$130"FREESTREAM VELOCITY m/s"
120 PRINT
130 INPUT TAB(15) U
140 PRINTCHR$130"LENGTH OF PLATE mm"
150 PRINT
160 INPUT TAB(15) LMAX
170 PRINTCHR$130"FREESTREAM TURBULENCE"
180 INPUT TAB(15) Tu
190 PRINT
200 PRINTCHR$130"DO YOU WANT TO USE ABU-GHANNAM&SHAHS"
210 PRINTCHR$130"CORRELATION FOR THE ONSET OF TRANSITION"
220 INPUT TAB(15) D$
230 IF D$= "Y" G0T0298
240 PRINT
250 PRINTCHR$130"INPUT TRANSITION ONSET IN mm"
260 INPUT TAB(15) Xst
270 PRINT
280 GOTO300
290 Xst=0
300 PRINTCHR$130"DO YOU WANT A PRINTOUT OF"
310 PRINTCHR$130"TRANSITION VELOCITY PROFILES"
320 INPUT TAB(15) PRO$
330 CLS
340 PRINT
350 PRINT
360 PRINTCHR$131"THE FREESTREAM VELOCITY DISTRIBUTION IS"
370 PRINTCHR$131"DEFINED IN THE FORM :-
380 PRINT
390 PRINTCHR$130;CHR$141; "U/UO = "CHR$141;"EX"CHR$140;"P;CHR$141;"* A + BX + CX";CHR$140;"* DX";CHR$140;"* "
400 PRINTCHR$130;CHR$141; "U/UO = ";CHR$141;"EX"CHR$140;" " ;CHR$141;"* A + BX + CX";CHR$140;" " ;CHR$141;"* DX"
410 PRINT
420 PRINT
430 PRINTTAB(4)CHR$134"P = "
440 INPUTTAB(10,10); P
450 PRINT
460 PRINTTAB(4)CHR$134"E = "
470 INPUTTAB(10,12); E
480 PRINT
490 PRINTTAB(4)CHR$134"A = "
500 INPUTTAB(10,14); A
510 PRINT
520 PRINTTAB(4)CHR$134"B = "
530 INPUTTAB(10,16); B
540 PRINT
550 PRINTTAB(4)CHR$134"C = "
560 INPUTTAB(10,18); C
570 PRINT
580 PRINTTAB(4)CHR$134"D = "
590 INPUTTAB(10,20); D
600 PRINT
610 E=4.28309
620 MODE4
630 DGDU=0:A2DSTR2=0
640 DIM THETA(100),DSTR1(100),X(100),CFL(100),H(100),S(100),DSTR2(100)
650 DIM Z(12),UN5(12),U(100),a(100)
660 DIM A(10,20),AN1V(10,10),C(10),B(10)
670 rho=(0.46535+2/(1+273))
680 mu=1.725+0.004375*1/10*5
690 nu=nu/rho
700 THETA(0)=0:DSTR1(0)=0;U(0)=UO:X(0)=0
710 CFL(0)=0;H(0)=0;G(0)=0:DSTR2(0)=0;a(0)=0
720 DI=10:SUM=0
730 I=1
740 DX=DI/10
750 GOTO730
760 IF(I+1)XMAX GOTO 650
770 I=I+1
780 X(I)=X(I-1)+DX
790 PROCQuadtrature
800 IF LAM=0 GOTO 1020
810
810 IF LAM<0.89GOT0820 ELSE 830
820 I=1:GOT0850
830 PROCtani
840 GOTO760
850 A1DSTR2=A1DSTR2/1
860 1DSTR2=0
870 IF ABS(A1DSTR2-A2DSTR2)<0.0001GOTO850
880 A2DSTR2=A1DSTR2
890 PROCcurvefit
900 I=1:GOT0940
910 I=I+1
920 IF I=N GOTO940 ELSE 930
930 GOTO760
940 DUDX=(X(I)/XHAX)*(U(I)*1000/XHAX)
950 LAH=DUDDU/XHAX
960 PROCtani
970 GOTO940
980 PROCPlot
990 IF LAM<0.89GOTO910 ELSE 1010
1010 IF SET=32 GOTO1020 ELSE 1010
1020 PROCput
1030 CHAIN^I6BLPR5*1040 END
1050 DEFPROCQuadrature
1060 LOCAL N,N1,N2
1070 X(I)/XHAX
1080 UN=E*SAP+A+B*B+C*SA2+D*0A3
1090 DUDX=(P*E*BA(P-1)+B+2*C*B+3*D*QA2)*(UO*100/XHAX)
1100 N=11
1110 ST=DX/10
1120 JST=X(I)-DX+ST
1130 FOR K=1TO N1 STEP2
1140 SUM=SUM+UN5(K)
1150 K=K+1
1160 FOR K=2 TO N1 STEP2
1170 SUM=SUM+4*UN5(K)
1180 NEXT
1190 N2=N-2
1200 FOR K=3 TO N2 STEP2
1210 SUM=SUM+2*UN5(K)
1220 NEXT
1230 N3=N-3
1240 FOR K=4 TO N3 STEP2
1250 SUM=SUM+3*UN5(K)
1260 NEXT
1270 INUN5=ST*SUM/(3*1000)
1280 U(I)=UN400
1290 THETA(I)=SQRT((.45+U05/U(I)+1)*INUN5)*1000
1300 LAM=DUDDU*THETA(I)*2/(nu*1000000)
1310 IF LAM<0.89GOTO1320 ELSE 1320
1320 I=I+1:GOT0850
1330 ENDPROC
1340 DEFPROCtani
1350 a=0
1360 TD=0.4-a/20
1370 TF=(776/5005)+(73/5005)*a-(23/5460)*a^2-(1/2860)*a^3
1380 TP=2*a*TE
1390 TQ=(4/35)*(TF)*(48-4*a+3*a^2)
1400 H=TD/TE
1410 S=TF/TE
1420 1=1:GOTO1340
1430 THETA(I)=THETA(I)+H(I)
1440 DSTR2(I)=THETA(I)*B(I)
1450 CSTR1(I)=TP*nu*1000/(THETA(I)*U(I))
1460 S1DSTR2=S1DSTR2+DSTR2(I)
1470 ENDPROC
1480 DEFPROCcurvefit
1490 N=1
1500 LOCALI
1510 M=2
1520 MAX=2*M
1530
REM INITIALISATION OF MATRICES

FOR I=1 TO M
FOR J=1 TO MAX
A(I,J)=0
NEXT
NEXT
A(1,1)=N
FOR I=2 TO H
FOR K=1 TO N
A(I,1)=A(I,1)+U(K)A(I-1)
NEXT
NEXT
FOR J=2 TO H
FOR K=1 TO N
A(H,J)=A(H,J)+U(K)A(H,J-2)
NEXT
NEXT
FOR J=2 TO H
FOR I=1 TO (H-1)
A(I,J)=A((I+1),(J+1))
NEXT
NEXT
FOR I=1 TO H
A(I,M+I)=1
B(I)=0
C(I)=0
NEXT
FOR I=1 TO M
FOR K=1 TO N
B(I)=B(I)+U(K)*A(I-1)
NEXT
NEXT
REM MATRICES FULLY INITIALIZED
L=1
FOR J=1 TO (M-1)
L=L+1
FOR I=L TO M
CONST=-A(I,J)/A(J,J)
FOR K=MAX TO J STEP -1
A(I,K)=A(I,K)+CONST*A(J,K)
NEXT
NEXT
L=0
FOR J=M TO 2 STEP -1
L=L+1
FOR I=(H-L) TO 1 STEP -1
CONST=-(A(I,J)/A(J,J))
FOR K=MAX TO 2 STEP -1
A(I,K)=A(I,K)+CONST*A(J,K)
NEXT
NEXT
FOR I=1 TO M
ANOR=A(I,I)
FOR J=1 TO MAX
A(I,J)=A(I,J)/ANOR
NEXT
NEXT
K=0
FOR J=(H+1) TO MAX
K=K+1
C(I)=C(I)+AINV(I,J)*B(J)
NEXT
NEXT
REM MATRIX INVERSION COMPLETE
ENDPROC
DEFPROCPlotGvU
FROCCurvefit
2478 Ctl=0
2480 VDU19,3,0,0,0
2490 MOVE 200,900:DRAW200,200:DRAW1200,200
2500 VDU5
2510 MOVE 100,000:PRINT"G"
2520 MOVE 1000,140:PRINT"U"
2530 VDU4
2540 Uscale=1000/ABS(U(1)-U(N))
2550 Gscale=700/0.1
2560 Ctl=0
2570 FOR K=1 TO N
2580 IF(U(K)<U(1)) GOTO2610
2590 Uplot=(U(K)-U(1))*Uscale+200
2600 GOTO2620
2610 Uplot=(U(K)-U(N))*Uscale+200
2620 Gplot=(G(K)-1.5)*Gscale+200
2630 PLOT(Uplot,Gplot)
2640 NEXT
2650 FOR K=1 TO N
2660 G=C(1)+C(2)*U(K)+C(3)*U(K)^2+C(4)*U(K)^3+C(5)*U(K)^4+C(6)*U(K)^5+C(7)*U(K)^6
2670 IF(U(K)<U(1)) GOTO2700
2680 Uplot=(U(K)-U(1))*Uscale+200
2690 GOTO2710
2700 Uplot=(U(K)-U(N))*Uscale+200
2710 Gplot=(G(K)-1.5)*Gscale+200
2720 IF(Ctl>0.5) GOTO2770
2730 Ctl=1
2740 MOVE Uplot,Gplot
2750 PLOT(Uplot,Gplot)
2760 GOTO2780
2770 DRAN(Uplot,Gplot)
2780 NEXT
2790 ENDP
2800 DEFPROC fplot
2810 CHX=QPENuUT,,TAND6DU*
2820 PRINT#CHX,C(1),C(2),C(3),C(4),z,t,XMAX,Tu,P,E,A,B,C,D,Xst,PRO#,Q#
2830 CLOSE#CHX
2840 ENDP
Programme

IGBLPR5

Programme IGBLPR5 is the main boundary layer prediction computational programme.
10 MODE1
20 VDU15
30 #T=20389
40 DIM THETA(350), DSTR1(350), X(350), Cf(350), H(350), B(350), DSTR2(350), U(350)
50 DIM Z(12), UNT(12)
60 DIM C(4), UNL(21), UNT(21), H(21)
70 DIM X(21), Y(21), DSTR2(12)
80 PROCread
90 rho=(0.46535*r/(t+273))
100 nu=0.725+0.004375*t/100
110 nu=nu/rho
120 THETA(0)=B: DSTR1(0)=U(0)=UO=X(0)=0
130 Cf(0)=B: H(0)=0: S(0)=B: DSTR2(0)=B: a=1.857: Ka=0.41: LAM=8
140 PRINT* X  Uinf  DSTR1  THETA  DSTR2  H12  H23  Cf*
150 PRINT
160 I=0
170 PROCprint
180 DX=25: SUM=0
190 IF X(I)>XMAX OR I>349 GOTO 670
200 I=I+1
210 X(I)=X(I-1)+DX
220 PROCtani
230 PROCstart
240 U(I)=UL
250 DSTR1(I)=DSTR1L
260 THETA(I)=THETAL
270 DSTR2(I)=DSTR2L
280 H(I)=HL
290 Cf(I)=CFL
300 PROCprint
310 IF X(I)>XMAX OR I>349 GOTO 670
320 IF X(I)>XMAX OR I>349 GOTO 670
330 IF X(I)>XMAX OR I>349 GOTO 670
340 GOTO360
350 IF TH > RTHS GOTO190
360 GOTO190
370 PRINT* I=1
380 X(I)=X(I-1)+DX
390 IF I>349 GOTO 370
400 IF X(I)>XMAX OR I>349 GOTO 370
410 VDU2
420 PROCtani
430 PROCAlber
440 PROCran
450 U(I)=UL
460 DSTR1(I)=DSTR1L
470 THETA(I)=THETAL
480 DSTR2(I)=DSTR2L
490 H(I)=HL
500 Cf(I)=CFL
510 LAM+THETA**2*dU/dx/(nu**100000)
520 PRINT* I=1
530 X(I)=X(I-1)+DX
540 IF I>349 GOTO 530
550 I=I+1
560 X(I)=X(I-1)+DX
570 IF I>349 GOTO 530
580 PROCAlber
590 U(I)=UT
600 DSTR1(I)=DSTRIT
610 THETA(I)=THETAT
620 DSTR2(I)=DSTR2T
630 H(I)=HT
640 Cf(I)=Cf
650 PROCprint
660 GOTO590
670 PRINT* PRINT: PRINT: PRINT
680 IF B$="Y" GOTO130
690 IF B$="GOTO730"
700 IF B$="GOTO730"
710 IF B$="GOTO730"
720 PRINT
730 END
740 DEFPRecip
750 IF P=10 THEN Q=350/58
760 IF P=10 THEN Q=350/58
770 IF P=10 THEN Q=350/58
780 IF P=10 THEN Q=350/58
790 IF P=10 THEN Q=350/58
800 PRINT* "YOU WANT TO PUT THIS DATA ON FILE"
BIB ENDPROC
820 DEFPROC Quadrature
830 LOCAL N,N1,N2
840 q=X(1)/XMAX
850 UNL=E*q+F+A+B*q+C*q^2+D*q^3
860 dUdX=(F+E*q*(P-1)+B+2*C*q+3*D*q^2)*(UO*1000/XMAX)
870 N=11
880 ST=DX/10
890 XST=X(I)-(DX+ST)
900 FOR K=1 TO N
910 XST=XST+ST
920 2(K)=XST/XHAX
930 UN5(K)=(E*Z(K)AP+A+B*Z(K)+C*Z(K)A2+D*2(K)A3)
940 NEXT
950 SUM=SUM+UN5(1)+UN5(11)
960 N1=N-1
970 FOR K=2 TO N1 STEP2
980 SUM=2*SUM+4*UN5(K)
990 NEXT
1000 N2=N-2
1010 FOR K=3 TO N2 STEP2
1020 SUM=3*SUM+2*UN5(K)
1030 NEXT
1040 INUNS=ST*SUM/(3*1000)
1050 UL=UNL*UO
1060 THETAL=SQR((0.45*nu*UO*A5/ULA6)*INUNS)*1000
1070 LAM=UO*THETAL^2/(nu*1000000)
1080 ENDPROC
1090 DEFPITani
1100 PROCUadrature
1110 IF LAM=0 GOTO 1260
1120 DGDU=C(2)+2*C(3)*UL+3*C(4)*UL^2
1130 a=0
1140 TD=0.4-a/20
1150 TE=(4/35)+(a/105)-12.27
1160 TF=(76/5005)+(73/5005)*a-(1-21/262)*a^2
1170 TP=2*a^2*TE
1180 TG=-(4/35)*TF+(46-4*a-3*a^2)
1190 HD=TD/TE
1200 G=TD/TE
1210 ita=(LAM*(H-I)-LAM*U)+DGDU*(G+TG/2)-a^2*(a/21)+a^2*(a/262)
1220 IF ABS(ita)>0.0016 GOTO1270
1230 =ita
1240 GOTO1140
1250 GOTO 1270
1260 a=1.857
1270 TD=0.4-a/20
1280 TE=(4/35)+(a/185)-12.27
1290 TF=(76/5005)+(73/5005)*a-(1-21/262)*a^2
1300 TP=2*a^2*TE
1310 HL=TD/TE
1320 Gl=TF/TE
1330 DSTRL=THETAL*HL
1340 DSTR=THETAL*Gl
1350 CFL=TP*nu*1000/(THETAL*UL)
1360 ENDPROC
1370 DEFPITread
1380 CHX=OPENIN"TAN3GDU"
1390 INPUT ;CHX,C(1),C(2),C(3),C(4),z,t,XMAX,Tu,UD,P,E,A,B,C,D,Xst,PRO$,G$
1400 CLOSECHX
1410 ENDPROC
1420 DEFPITstart
1430 IF G$="N" GOTO 1530
1440 IF LAM=0 GOTO 1470
1450 LAM=FLAM-6.91+12.75*LAM+6.64*LAM^2
1460 GOTO1480
1470 LAM=FLAM-6.91+2.48*LAM-12.27*LAM^2
1480 RTHS=163*EXP(FLAM-(FLAM*Tu/6.91))
1490 GOTO 1510
1500 RTHS=UL*1st/(nu*1000)
1510 RTH=THETAL*UL/(nu*1000)
1520 IF RTH>RTHS GOTO 1570
1530 IF X(I)<Xst GOTO 1470
1540 X=px*XMAX
1550 Ust=(E*X+P)+A+Bq+Cq*2+D*q^3+UO
1560 RTHS=THETAL*Ust/(nu*1000)
1570 US=UL*LAM=LAM:XS=X(I):THETAS=THETAL
1580 PRINT
1590 PRINTTAB(30)*START OF TRANSITION*
1600 PRINTTAB(31)*RTHAS = *RTHS
1610 IF LAM=0 GOTO 1640
1620 RS1=0.27-(0.25*TU3.5/(1+TU3.5))
1630 RS2=1/(1+1710*+LAM1.4*EXP(-SOR1+T3.5))
1648 RSig=RS1*RS2*1000000
1650 GOTO1670
1668 RSig=(0.27-(0.25*TuA3.5/<1+TuA3.5))*1000000
1670 Sigca=RSig*nu*1000/US
1680 XBAR=2.25*Sigca+XS
1690 eta=-2.25
1700 ENDPROC
1710 DEFPROCTran
1720 LOCAL n
1730 BSUf1=0:CSUM=0
1740 q=(X(i)/MAX)
1750 UNt=E4*q+A+B*q+Cq^2+D*q^3
1760 Ut=UNt*UO
1770 IF LAH<=0.6 GOTO1800
1780 PLAM=70*LAM+310*LAH^2-5430*LAH^3+66800*LAH^4
1790 6OTO1810
1800 PLAH=73*LAH+109*LAH^2+709*LAH^3
1810 eta=(X(i)-XBAR)/Sigaa
1820 Heta=SBR(eta^2)
1830 IF eta=0 GOTO 1880
1840 CL=0.8273*Meta*C2=0.894*Meta^2
1850 C3=0.073*Meta^3*C4=0.0185*Meta^4
1860 Gam=0.5*(1+((eta/Meta)^2)-(C1-C2-C3+C4))
1870 GOTO1890
1880 Gam=0.5
1890 DL=63*THETAL/(1.4-(PLAH/15)-(PLAH^2/144))
1900 DT=DSTT*Ka/(1+Pi)*f)
1910 n=2/(DH-1)
1920 IF BL=1950 GOTO 1930
1930 DEL=DT
1940 GOTO1960
1950 DEL=DL
1960 N=2
1970 DY=0.1
1980 FOR K=2 TO N
1990 Y=Y+DY
2000 IF Y>=DL GOTO 2070
2010 Lpr1=(2*Y/DL-2*(Y/DL)^3+(Y/DL)^4)
2020 Lpr2=PLAM*(Y/DL)^3-(Y/DL)^3-(Y/DL)^4)/6
2030 unL(K)=Lpr1+Lpr2
2040 unT(K)=(Y/DL)^2
2050 unL(K)=1.0
2060 unT(K)=1.0
2070 unL(K)=1.0
2080 unT(K)=1.0
2090 unL(K)=1.0
2100 unT(K)=1.0
2110 unK(K)=1.0
2120 unL(K)=1.0
2130 unK(K)=1.0
2140 IN2(K)=1-unL(K)*unT(K)
2150 UNL(K)=Y/DEL
2160 NEXT
2170 BSUM=1.0+4*IN1(21)
2180 CSUM=1.0+4*IN2(21)
2190 N1=N-1
2200 FOR K=2 TO N1 STEP 2
2210 BSUM=BSUM+4*IN1(K)
2220 CSUM=CSUM+4*IN2(K)
2230 NEXT
2240 N2=N-2
2250 FOR K=3 TO N2 STEP 2
2260 BSUM=BSUM+2*IN1(K)
2270 CSUM=CSUM+2*IN2(K)
2280 NEXT
2290 Qt=DY*BSUM/3
2300 Qt=2*DY*CSUM/3
2310 DSTT=DSRT1*(1-Gam)+DSRT2*Gam
2320 THI=THI*(1-Gam)+THETAL-Gam*THETAL+H1
2330 TH2=Gam*THETAT-(1-Gam)+THETAT*H2
2340 TH3=2-Gam*(1-Gam)*H1
2350 THETAT=THI+TH2+TH3
2360 DS2t1=(1-Gam)*((1-Gam)^2*DS2T1*Gam+(1-Gam)^2*DS2T1)
2370 DS2t2=Gam*(Gam^2*DS2T2+(Gam^2-1)*DSRT)
2380 DS2t3=Gam*(1-Gam+2*Gam-1)*DSRT
2390 DS2t4=Gam*(1-Gam-2)*DS2t2+DS2t3
2400 Ht=DSRT2/THETAT
2410 Gt=DSRT2/THETAT
2420 Cft=(1-Gam)*GAM^2*GAM^2
2430 IF PROD="Y"GOTO2480ELSE2530
2440 IFX=GETINT(IHR) GOTO 2530
2450 IF XPRONT(IHR) GOTO 2530
2460 PRINT*DELTA = " DEL;" " Gam = "; Gam = ";Gam=PRINT;PRINT
2470 PRINTTAB(10)"Velocity profile @ X = \";X(I);\" \n"
2480 PRINT:PRINT
2490 PRINT:Y/DELTA U/Unif*
2500 FOR K = 1 TO N
2510 PRINTTAB(10);YD(K);TAB(20);unt(K)
2520 NEXT
2530 ENDPROC
2540 DEFPROCAlber
2550 X=X(I)-DX
2560 PROCRunge
2570 DS=f*(1+Pi)/Ka
2580 TS=DS-fA2*(2+3.179*Pi+1.5*PiA2)/(KaA2)
2590 ST=3*TS-DS+fA3*(6+11.14*Pi+8.5*PiA2+2.56*PiA3)/(KaA3)
2600 CfT=(2*fA2)t(l+0.835*Tu)
2610 HT1=DS/TS:BT=ST/TS
2620 HT=HT1*(1-0.02*Tu)
2630 DSTR1=Dstr
2640 THETA1=DSTR1/HT1
2650 THETA=THETA1/(1.085*Tu)
2660 DSTR2=HT*THETA
2670 DSTR2=DSTR1/ST
2680 q=X/XMAX
2690 UNT=(E*qAP)+A+B*q+C*qA2+D*qA3
2700 UT=UNT*UQ
2710 dUdX=(UO*1000/XMAX)*((P*E*qAP-P-1)+B+2*C*q+3*D*qA2
2720 DSTR=DSTR1/D1
2730 DSTR=DSTR1/D1
2740 DSTR=DSTR1/D1
2750 F1=DSTR+DSTR+DSTR+DSTR/F1
2760 F1=F1+DSTR+DSTR+DSTR+DSTR/F1
2770 K1=F1*(1+GH)/(1+GH)
2780 K1=F1*(1+GH)/(1+GH)
3300 \( CfT = 0.3/(\exp(1.33*H) \times (0.434294 \times \ln(\Theta))^{1.74+0.31*H}) \)
3310 \( f = \sqrt{CfT}/2 \)
3320 \( \beta_{TA} = -D5_{tr} \times dU/dx/(f^2 \times U(I) \times 10^{B0}) \)
3330 \( P_i = 0.8 \times (0.5 + \beta_{TA})^{0.75} \)
3340 ENDPROC
3350 DEFPORCFput
3360 N=1
3370 PRINTTAB(7,12)CHR$134"INPUT NAME OF FILE"
3380 INPUT TAB(15) PRED$
3390 PRINT;PRINT;
3400 PRINTTAB(I)CHR$134"ON WHICH DRIVE IS FILE TO BE STORED"
3410 BZ=SET
3420 PROCDrive(BZ)
3430 CHX=OPENOUT(PRED$)
3440 PRINTCHX,N,XMAX
3450 FOR I = 1 TO N
3460 PRINTCHX,DSTR1(I),THETA(I),DSTR2(I),Cf(I),X(I),U(I)
3470 NEXT
3480 CLOSE# CHX
3490 ENDPROC
3500 DEFPORCDrive(BZ)
3510 IF BZ=48 THEN *DR.0
3520 IF BZ=49 THEN *DR.1
3530 IF BZ=50 THEN *DR.2
3540 IF BZ=51 THEN *DR.3
3550 ENDPROC
Programme

GRAFPC3

Programme GRAFPC3 is a graphics programme which can be used to display the predicted development of the boundary layer parameters and to compare the predictions with experimental data held on a data file.
10 REM BOUNDARY LAYER PREDICTION GRAPHICS PACKAGE
20 CT=B:G0T23="Y"
30 CT=B:G0T24="N"
30 DIM DSTR1(301),DSTR2(301), THETA(301)
40 DIM CF(301), X(301), U(301)
50 DIM EDSTR1(30), ETHETA(30), EDSTR2(30)
60 DIM ECI(30), EHI2(30), EHI3(30), EI130)
70 MODE 7
80 PROC Intro
90 MODE 4
100 PROCAxes
110 IF G3$="*N" G0T0 130
120 PROC Fread
130 IF G4$="*N" G0T0 160
140 PROC Fread
150 IF G3$="*N" G0T0 170
160 PROC DSTR1
170 IF B$="*N" G0T0 190
180 PROC EX DSTR1
190 IF B2$="*N" G0T0 310
200 IF Ct=1 G0T260
210 VDU4:PRINTTAB(9,1)"PRESS SPACE TO CONTINUE"
220 PRINTTAB(10,3)"NEXT GRAPH THETA vs X"
230 B=GET
240 CLS
250 IF B<>32 G0T0 210
260 Ct=1
270 GCOL 0,0: G0T0 150
280 B=GET
290 GCOL 0,1
300 IF G3$="*N" G0T0 320
310 PROC THETA
320 IF B$="*N" G0T0 340
330 PROC ETHETA
340 IF B2$="*N" G0T0 360
350 G0T0 160
360 VDU4:PRINTTAB(9,1)"PRESS SPACE TO CONTINUE"
370 PRINTTAB(10,3)"NEXT GRAPH DSTR1 vs X"
380 B=GET
390 CLS
400 IF B<>32 G0T0 380
410 Ct=1
420 GCOL 0,0: G0T0 300
430 Ct=0:GCOL 0,1
440 IF G3$="*N" G0T0 460
450 PROC DSTR2
460 IF B$="*N" G0T0 480
470 PROC EX DSTR2
480 IF B2$="*N" G0T0 500
490 Ct=1: G0T0 520
500 VDU4:PRINTTAB(9,1)"PRESS SPACE TO CONTINUE"
510 PRINTTAB(10,3)"NEXT GRAPH CF vs X"
520 B=GET
530 CLS
540 IF B<>32 G0T0 540
550 Ct=1
560 GCOL 0,0: G0T0 440
570 Ct=0:GCOL 0,1
580 IF G3$="*N" G0T0 680
590 PROC CF
600 IF B$="*N" G0T0 620
610 PROC EX CF
620 IF B2$="*N" G0T0 640
630 GCOL 0,1
640 IF G3$="*N" G0T0 660
650 PROC H12
660 IF B$="*N" G0T0 700
670 PROC EX H12
680 IF B2$="*N" G0T0 720
690 IF G3$="*N" G0T0 740
700 PROC H12
710 IF G3$="*N" G0T0 760
720 PROC EX H12
730 IF Ct=1 G0T780
740 IF B$="*N" G0T0 790
750 IF B2$="*N" G0T0 810
760 IF G3$="*N" G0T0 830
770 IF Ct=1 G0T0 850
780 VDU4:PRINTTAB(9,1)"PRESS SPACE TO CONTINUE"
790 PRINTTAB(10,3)"NEXT GRAPH H12 vs X"
800 B=GET
810 CLS
820 IF B<>32 G0T0 820
838 Cla=1
840 DCOLB,0:GOTO720
850 Cla=B:DCOLB,1
860 IF Q3$="N" GOTO 880
870 PR0CH32
880 IF Q$="N" GOTO 900
890 PR0CEXH32
900 IF Q2$="Y"60TQ 970
910 CLS
920 IF Q3$="N" GOTO 980
930 PRINTTAB(10,1)'DO YOU WANT PRINTOUT *
940 PRINTTAB(6,3)"OF ALL GRAPHS COMBINED Y/N"
950 INPUT TAB(20) Q2$
960 IF Q2$="Y"G0T016B
978 VDU16
988 END
998 DEFPRQCAxes
1088 VDU2B,8,5,39.0
1098 VDU19.3,3.0.0.0
1020 MOVE 225,750:DRAW 225,150
1030 DRAW 1125,150
1040 MOVE 1150,150:DRAW 1150,750
1050 VDU5
1060 MOVE200,250:DRAW225,250
1070 MOVE170,268:PRINT*1
1080 MOVE200,350:DRAW225,350
1090 MOVE170,368:PRINT*2
1100 MOVE200,450:DRAW225,450
1110 MOVE170,468:PRINT*3
1120 MOVE200,550:DRAW225,550
1130 MOVE170,568:PRINT*4
1140 MOVE200,650:DRAW225,650
1150 MOVE170,668:PRINT*5
1160 MOVE200,750:DRAW225,750
1170 MOVE170,768:PRINT*6
1180 MOVE,780:PRINT*7
1190 MOVE,625:PRINT*8
1200 MOVE,550:PRINT*9
1210 MOVE,475:PRINT*10
1220 MOVE1175,350:DRAW1150,350
1230 MOVE1175,368:PRINT*11
1240 MOVE1175,550:DRAW1150,550
1250 MOVE1175,568:PRINT*12
1260 MOVE1175,750:DRAW1150,750
1270 MOVE1175,768:PRINT*13
1280 MOVE1190,675:PRINT*H12
1290 MOVE1190,688:PRINT*H12
1300 ENDPROC
1310 DEFPROCIntro
1320 PRINTTAB(6.4)CHR$134"DO YOU HAVE A PREDICTED"
1330 PRINTTAB(12)CHR$134"DATA FILE Y/N"
1340 INPUT TAB(20) Q3$
1350 PRINT:PRINT:PRINT
1360 IF Q3$="N" GOTO1398
1370 PRINT:PRINT
1380 IF Q3$="N" GOTO1398
1390 PRINT:PRINT:PRINT
1400 PRINTTAB(6)CHR$134"DO YOU HAVE AN EXPERIMENTAL"
1410 PRINTTAB(12)CHR$134"DATA FILE Y/N"
1420 INPUT TAB(20) B$
1430 IF B$="N" GOTO 1470
1440 PRINT
1450 PRINTTAB(4)CHR$134"NAME OF EXPERIMENTAL DATA FILE*"
1460 INPUT TAB(16) EXDAT$
1470 ENDPROC
1480 IF Q$="Y" GOTO 1520
1490 PRINTTAB(16)CHR$134"INPUT XMAX"
1500 INPUT TAB(20) XMAX
1510 PRINT
1520 PRINTTAB(7)CHR$134"WHICH DRIVE ARE FILES ON*"
1530 B=7:SET
1540 PRODrive(B)
1550 ENDPROC
1560 DEFPROCbRead
1570 CHX=OPENIN(PRED$
1580 INPUT#CHX,N,XMAX
1590 FOR I=1 TO N
1600 INPUT#CHX,DSTR1(I),THETA(I),DSTR2(I),Cf(I),X(I),U(I)
1610 NEXT
1620 CLOSE#CHX
1630 ENDPROC
1640 DEFPROCDSTRI
1650 IFB2$="Y"GOTO1670

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1660 VDU5: MOVE 400, 600: PRINT "DSTR1 vs X/XMAX": VDU4
1670 DS1P=DSTR1(1)*100+150
1680 XP=(X(1)/XMAX)*900+225
1690 DRAW XP, DS1P
1700 FOR I = 1 TO N
1710 DS1P=DSTR1(1)*100+150
1720 XP=(X(I)/XMAX)*900+225
1730 DRAW XP, DS1P
1740 NEXT
1750 ENDPROC
1760 DEFPROC THETA
1770 IF Q2$='Y' GOTO 1790
1780 VDU5: MOVE 400, 800: PRINT "THETA vs X/XMAX": VDU4
1790 THP=THETA(1)*100+150
1800 XP=(X(1)/XMAX)*900+225
1810 MOVE XP, THP
1820 FOR I = 1 TO N
1830 THP=THETA(I)*100+150
1840 XP=(X(I)/XMAX)*900+225
1850 DRAW XP, THP
1860 NEXT
1870 ENDPROC
1880 DEFPROCDSTR2
1890 IF Q2$='Y' GOTO 1910
1900 VDU5: MOVE 400, 600: PRINT "DSTR2 vs X/XMAX": VDU4
1910 DS2P=DSTR2(1)*100+150
1920 XP=(X(1)/XMAX)*900+225
1930 MOVE XP, DS2P
1940 FOR I = 1 TO N
1950 DS2P=DSTR2(I)*100+150
1960 XP=(X(I)/XMAX)*900+225
1970 DRAW XP, DS2P
1980 NEXT
1990 ENDPROC
2000 DEFPROCF
2010 IF Q2$='Y' GOTO 2030
2020 VDU5: MOVE 450, 800: PRINT "CF vs X/XMAX": VDU4
2030 CFP=CF(I)*100*100+150
2040 XP=(X(1)/XMAX)*900+225
2050 MOVE XP, CFP
2060 FOR I = 2 TO N
2070 CFP=CF(I)*100*100+150
2080 XP=(X(I)/XMAX)*900+225
2090 DRAW XP, CFP
2100 NEXT
2110 ENDPROC
2120 DEFPROCH12
2130 IF Q2$='Y' GOTO 2150
2140 VDU5: MOVE 450, 800: PRINT "H12 vs X/XMAX": VDU4
2150 HP=DSTR1(1)/THETA(1)*200+150
2160 XP=(X(1)/XMAX)*900+225
2170 MOVE XP, HP
2180 FOR I = 1 TO N
2190 HP=DSTR1(I)/THETA(I)*200+150
2200 XP=(X(I)/XMAX)*900+225
2210 DRAW XP, HP
2220 NEXT
2230 ENDPROC
2240 DEFPROCH32
2250 IF Q2$='Y' GOTO 2270
2260 VDU5: MOVE 450, 800: PRINT "H32 vs X/XMAX": VDU4
2270 GP=DSTR2(1)/THETA(1)*200+150
2280 XP=(X(1)/XMAX)*900+225
2290 MOVE XP, GP
2300 FOR I = 1 TO N
2310 GP=DSTR2(I)/THETA(I)*200+150
2320 XP=(X(I)/XMAX)*900+225
2330 DRAW XP, GP
2340 NEXT
2350 ENDPROC
2360 DEFPROCXDSTR1
2370 FOR I = 1 TO N1
2380 DS1P=DOSTR1(I)*100+150
2390 XP=EX(I)/XMAX*900+225
2400 PLOT69, XP, DS1P
2410 NEXT
2420 ENDPROC
2430 DEFPROCXTHETA
2440 FOR I = 1 TO N1
2450 THP=ETHETA(I)*100+150
2460 XP=EX(I)/XMAX*900+225
2470 PLOT69, XP, THP
2480 NEXT
2490 ENDPROC
2500 DEFPROC1DSTR2
2510 FOR I=1 TO N1
2520 DS2=EDSTR2(11*100+150
2530 XP=EX(I)/XMAX*900+225
2540 PLOT69,XP,DS2
2550 NEXT
2560 ENDPROC
2570 DEFPROC2X:
2580 FOR I=1 TO N1
2590 CF=ECf(11*100+150
2600 XP=EX(I)/XMAX*900+225
2610 PLOT69,XP,CF
2620 NEXT
2630 ENDPROC
2640 DEFPROCEXH12
2650 FOR I=1 TO N1
2660 HP=EDSTR1(I)/ETHETA(I)*200+150
2670 XP=EX(I)/XMAX*900+225
2680 PLOT69,XP,HP
2690 NEXT
2700 ENDPROC
2710 DEFPROCEXH32
2720 FOR I=1 TO N1
2730 GP=EDSTR2(I)/ETHETA(I)*200+150
2740 XP=EX(I)/XMAX*900+225
2750 PLOT69,XP,GP
2760 NEXT
2770 ENDPROC
2780 DEFPROCRead
2790 chX=OPENIN(EXDAT$)
2800 INPUT§chX,N1,Tu,UO
2810 FOR I=1 TO N1
2820 INPUT§chX,EX(I),EDSTR1(I),ETHETA(I),EDSTR2(I),ECf(I)
2830 NEXT
2840 CLOSE§chX
2850 ENDPROC
2860 DEFPROCDrive(BZ)
2870 IF BZ=48 THEN *DR.0
2880 IF BZ=49 THEN *DR.1
2890 IF BZ=50 THEN *DR.2
2900 IF BZ=51 THEN *DR.3
2910 ENDPROC